Performance Evaluation of Elliptical and Multi-part Mirrors for Concentrating Diffused Light onto Collector Surface from Half Celestial Sphere

MITSUHIRO MATSUMOTO^{*†} Member

(Received February 3, 2021, revised July 19, 2021)

Abstract: An elliptical mirror has a mirror surface at the first focal point on a plane perpendicular to its central axis, which, in turn, passes through the first and second focal points. It can collect diffused light from a half celestial sphere, which extends 360° in the axial rotation direction and 180° in the axial direction at the second focal point. An elliptical mirror allows only the light incident on the first focal point to be concentrated on only the second focal point. A single elliptical mirror consists of only one elliptical mirror. A multi-part mirror is composed of multiple elliptical mirrors. In a multi-part elliptical mirror, the first focal point lies on its surface, and the second focal point is so positioned that it is aligned with the top of its surface. A multi-part mirror can concentrate diffused light from a wider area than a single elliptical mirror because its first focal points are dispersed. Furthermore, in a single elliptical mirror, only the light incident on the first focal point is concentrated on only the second focal point; therefore, the light-concentrating efficiency is low. In this study, we installed a collector surface on the second focal point of an elliptical mirror to increase its light-concentrating efficiency. Subsequently, we evaluated single elliptical and multi-part mirrors that capture the diffused light from a half celestial sphere not incident on the first focal point and concentrate it on the collector surface as well as the second focal point. We configured a coordinate system for the design of single elliptical and multi-part mirrors and subsequently analyzed the mechanism by which the light will strike these designed mirrors and concentrate on their collector surfaces. Based on the design values, we manufactured actual single elliptical and multi-part mirrors, which were illuminated with diffused light, and measured the illuminance of the light concentrated on their collector surfaces. Finally, we evaluated the light-concentrating performance of these mirrors based on the analysis results and measured illumination.

Keywords: Elliptical Mirror, Multi-part Mirror, Light Concentrating, Diffused Light, Collector Surface, Half Celestial Sphere, Illuminance, Illuminance Ratio

1. Introduction

Light-concentrating collectors, which employ a concentrator to concentrate direct sunlight and convert this concentrated light energy into heat energy by emitting it onto a heat collector at the collector surface are widely used [1]. Plane mirrors, concave mirrors, composite parabolic mirrors, and Fresnel lenses are used as concentrators [1]. However, a light-concentrating collector only concentrates direct light from the sun and cannot concentrate diffused light from the entire sky, which is a half celestial sphere [2]. The proportion of diffused light is high in most parts of the world, except in desert areas where the weather is sunny all year round [2]. In Japan, approximately half of the total annual solar radiation is diffused light [2]. In particular, on the Sea of Japan side from San'in to Hokkaido, there is an average cloud cover of 8.5 or more for more than 180 days during

which only diffused light irradiates [3]. Therefore, locations such as the Sea of Japan side of Japan require a concentrator that can concentrate the diffused light from a half celestial sphere on the collector surface.

A variety of research has been conducted on concentrators for concentrating diffused light. Winston, Derrick et al., and Rao et al. presented composite parabolic mirrors arranged in parallel [4]~[6]. Mather et al. and Mills et al. presented symmetric and asymmetric cylindrical mirrors [7][8]. Mather et al. and Derrick et al. proposed an involute-shaped mirror [5][7]. Bassett et al. suggested mirrors with symmetrical and asymmetric shapes [9]. Grimmer, Frissora et al., Norton et al., Suzuki, Eames et al., and Wang et al. presented composite parabolic mirrors possessing symmetric and asymmetric shapes [10]~[16]. Grimmer and Derrick et al. presented a parabolic mirror [5][10]. Derrick et al. recommended a plane mirror [5]. Yamada et al. proposed Fresnel lenses, cylindrical lenses, water cylinder lenses, and parabolic mirrors [17]. However, all these studies suggested two-dimensional concentrators, which are

^{*} Corresponding: m-matsumoto@kanagawa-u.ac.jp

[†] Faculty of Engineering, Kanagawa University, 3-27-1, Rokkakubashi, Kanagawa-ku, Yokohama-shi, Kanagawa, 221-8686, Japan

Table 1: List of variables used in equations.

Variable	Meaning	Variable	Meaning
a	Variable in Eq. (15)	r_{Fj0}	Coordinate of F_{j0}
b	Variable in Eq. (15)	r_{F00}	Coordinate of F_{00}
e	Illuminance ratio	r_{jk}	Coordinate in $R_{jk}-\Theta_{jk}-Z_{jk}$
f	Focal length of the ellipse	r_{Rm}	Coordinate of R_m
ϕ_{Nm}	Angle between normal of ellipse at R_m and R_{jk}	r_{Rm-1}	Coordinate of R_{m-1}
ϕ_{Rm}	Angle between reflected light at R_m and R_{jk}	r_{R0}	Coordinate of R_0
ϕ_{Rm-1}	Angle between reflected light at R_{m-1} and R_{jk}	r_{00}	Coordinate in $R_{00}-\Theta_{00}-Z_{00}$
ϕ_{R0}	Angle between incident light at R_0 and R_{jk}	θ_{Fj}	Angle from F_{j0} to F_{j1}
g	Distance from edge to another edge of mirror surface	θ_{Fjk}	Coordinate of F_{jk}
h	Height of the elliptical mirror	θ_{Fj0}	Coordinate of F_{j0}
i_D	Illuminance of the dome light	θ_{F00}	Coordinate of F_{00}
i_M	Illuminance of the measured object	ξ_F	Angle from F_{00} to F_{10}
i_S	Illuminance of the light-shielded measurement object	ξ_{Fj}	Angle from $R_{00}-\Theta_{00}-Z_{00}$ to $R_{j0}-\Theta_{j0}-Z_{j0}$
i_T	Illuminance of the only measured object	z	Coordinate in $R-\Theta-Z$
j	Number indicating elliptical mirror in the R-axis	z_{Fjk}	Coordinate of F_{jk}
k	Number indicating elliptical mirror in the Θ -axis	z_{Fj0}	Coordinate of F_{j0}
m	Number of reflections	z_{F00}	Coordinate of F_{00}
n_M	Number of all elliptical mirrors	z_{jk}	Coordinate in $R_{jk}-\Theta_{jk}-Z_{jk}$
n_R	Number of elliptical mirrors in the R-axis	z_{Rm}	Coordinate of R_m
$n_{\Theta j}$	Number of elliptical mirrors in the Θ -axis	z_{Rm-1}	Coordinate of R_{m-1}
r	Coordinate in $R-\Theta-Z$	z_{00}	Coordinate in $R_{00}-\Theta_{00}-Z_{00}$
r_C	Coordinate of C	$2l$	Length of the major axis of ellipse
r_E	Radius of the elliptical mirror	$2s$	Length of the minor axis of ellipse
r_{Fjk}	Coordinate of F_{jk}	-	-

inadequate for concentrating the diffused light from a three-dimensional half celestial sphere. Other studies have been conducted on three-dimensional concentrators. Sellami et al. presented a square elliptical hyperbolic mirror [18]. However, it emphasized the injection of direct light into a solar cell.

Matsumoto presented a multi-part mirror arranged with an array of elliptical mirrors [19]. An elliptical mirror has a mirror surface at the first focal point on a plane perpendicular to its central axis, which, in turn, passes through the first and second focal points. It can collect diffused light from a half celestial sphere that extends 360° in the axial rotation direction and 180° in the axial direction at the second focal point. An elliptical mirror allows only the light incident on the first focal point to be concentrated on only the second focal point. A multi-part mirror is composed of multiple elliptical mirrors. In all the elliptical mirrors of a multi-part mirror, the first focal point is located on the surface of the multi-part mirror, and the second focal point is positioned so that it is aligned to its top. A multi-part mirror can concentrate diffused light from a wider area than a single elliptical mirror because the first focal points of the former are dispersed on its surface. However, in a multi-part mirror, only the light incident on the first focal point is concentrated on only the second focal point; therefore, the light-concentrating efficiency is low.

In this study, we installed a collector surface on the second focal point of an elliptical mirror, to increase its light-concentrating efficiency. Subsequently, we evaluated single elliptical and multi-part mirrors that capture the diffused light from a half celestial sphere not incident on the first

focal point and concentrate it on the collector surface as well as the second focal point. We configured a coordinate system for the design of single elliptical mirrors and multi-part mirrors and subsequently analyzed the mechanism by which the light is incident on our designed single elliptical and multi-part mirrors and concentrates on their collector surfaces. Based on the design values, we manufactured actual single elliptical and multi-part mirrors. Using our experimental set-up, we illuminated the manufactured mirrors with diffused light and measured the illuminance of the light concentrated on their collector surfaces. Finally, we evaluated the light-concentrating performance of these single elliptical and multi-part mirrors based on the analysis results and the measured illumination.

2. Design

2.1 Elliptical Mirror Table 1 lists the variables in the equations used in this study. We set up the coordinate system of an elliptical mirror to design single elliptical mirrors and elliptical mirrors for multi-part mirrors. Figure 1 presents the elliptical mirror coordinate system and design method. Figures 1(a) and (b) show the front and longitudinal views, respectively. In the coordinate system, F_{jk} , and the origin, O_{jk} , are the first and second focal points of the elliptical mirror, respectively. The central axis of the elliptical mirror passes through the first focal point, F_{jk} , and the origin, O_{jk} . The coordinate system of the elliptical mirror is $R_{jk}-\Theta_{jk}-Z_{jk}$, where the R_{jk} axis is the radial direction, Θ_{jk} axis is the direction of rotation around the center axis, and Z_{jk} axis is the center axis direction. The elliptical mirrors are indexed as j and k . Coordinates in the $R_{jk}-\Theta_{jk}-Z_{jk}$ co-

Figure 1: Coordinate system and design method of elliptical mirror at front view and longitudinal view.

Figure 2: Coordinate system and design method of single elliptical and multi-part mirrors at front view and longitudinal view.

ordinate system are denoted as $(r_{jk}, \theta_{jk}, z_{jk})$.

An elliptical mirror has a three-dimensional shape obtained by rotating an ellipse around the Z_{jk} axis on the R_{jk} -

Z_{jk} plane in the R_{jk} - Θ_{jk} - Z_{jk} coordinate system, which allows axisymmetry to be achieved with respect to the Z_{jk} axis. The focal length, length of the major axis, and length of the minor axis of the ellipse are denoted as f , $2l$, and $2s$, respectively. The mirror surface is at the first focal point, F_{jk} , on a plane perpendicular to the Z_{jk} axis and parallel to the R_{jk} - Θ_{jk} plane in the R_{jk} - Θ_{jk} - Z_{jk} coordinate system passing through the first focal point, F_{jk} . The radius and height of the elliptical mirror are denoted as r_E and h , respectively.

The range of the light incident on the elliptical mirror is a half celestial sphere from the first focal point, F_{jk} , in the direction of the origin, O_{jk} . In the design of the elliptical mirror, focal length f , radius r_E , and height h are determined using the equations below. The focal length, f , can be expressed as

$$f = \sqrt{l^2 - s^2} \quad (l > s > 0) \quad (1)$$

Using Eq. (1), the equation of the ellipse can be expressed as

$$\frac{r_{jk}^2}{s^2} + \frac{(z_{jk} + 2f)^2}{l^2} = 1 \quad (z_{jk} \leq -2f) \quad (2)$$

Using Eq. (2), the radius, r_E , can be expressed as

$$r_E = \frac{s^2}{l} \quad (3)$$

Using Eq. (1), the height, h , can be expressed as

$$h = l - f \quad (4)$$

2.2 Single Elliptical Mirror and Multi-part Mirrors

We set up a coordinate system for designing single elliptical and multi-part mirrors. Figure 2 shows the coordinate system and design method for these single elliptical and multi-part mirrors. Figures 2(a) and (b) present the front and longitudinal views, respectively. A single elliptical mirror has a first focal point F_{00} . A multi-part mirror is composed of multiple elliptical mirrors with multiple first focal points, F_{jk} . For each elliptical mirror of a multi-part mirror, the first focal point, F_{jk} , lies on the surface of the multi-part mirror. The elliptical mirrors are positioned on the surface of the multi-part mirror. Adjacent elliptical mirrors are arranged with a certain distance between them. The origin, O_{jk} , which is the second focal point of all the elliptical mirrors, is aligned with the top of the multi-part mirror surface.

In the coordinate system, the origin, O , denotes the second focal point of all the elliptical mirrors, and the central axis of a single elliptical or multi-part mirror passes through this point. The coordinate system is defined as R - Θ - Z , where R axis is the radial direction, Θ axis is the direction of rotation around the center axis, and Z axis is the center axis direction. Coordinates in the R - Θ - Z coordinate system are denoted as (r, θ, z) . For an elliptical mirror with a first focal point F_{00} and origin O_{00} , the R_{00} , Θ_{00} , and Z_{00} axes in the R_{00} - Θ_{00} - Z_{00} coordinate system coincide with the origin, O , and the R , Θ , and Z axes in the R - Θ - Z coordinate system. Coordinates in the R_{00} - Θ_{00} - Z_{00} coordinate system are denoted as $(r_{F00}, \theta_{F00}, z_{F00})$.

For an elliptical mirror having a first focal point F_{j0} , the R_{j0} - Θ_{j0} - Z_{j0} coordinate system is the coordinate system obtained by rotating the R_{00} - Θ_{00} - Z_{00} coordinate system at an angle of ξ_{Fj} about the origin, O_{00} , on the R_{00} - Z_{00} plane. The index, j , denotes the number of elliptical mirrors rotated accordingly, where j is a natural number including 0. In this rotation, g denotes the edge-to-edge distance on the mirror surface from the first focal point, F_{00} , to first the focal point of an adjacent elliptical mirror, F_{10} , and ξ_F denotes the corresponding angle. Angle ξ_F is found using trigonometry based on the geometry of angle ξ_F , focal length f , radius r_E , and distance g , which are shown in Fig. 2(b). For this rotation, the number of elliptical mirrors is denoted as n_R . Coordinates in the R_{j0} - Θ_{j0} - Z_{j0} coordinate system are denoted as $(r_{Fj0}, \theta_{Fj0}, z_{Fj0})$.

For an elliptical mirror with a first focal point F_{jk} , the R_{jk} - Θ_{jk} - Z_{jk} coordinate system is defined as a coordinate system rotated at an angle θ_{Fjk} about the Z_{00} axis centered in the R_{j0} - Θ_{j0} - Z_{j0} coordinate system. Index k denotes the number of elliptical mirrors rotated accordingly, where k is a natural number including 0. In this rotation, θ_{Fj} denotes the angle from the first focal point, F_{j0} , to the first focal point of an adjacent elliptical mirror, F_{j1} . Angle θ_{Fj} is found using trigonometry based on the geometry of angle θ_{Fj} , radius r_E , and radius r_{Fj0} , which are presented in Fig. 2(a). For this rotation, the number of elliptical mirrors is denoted as $n_{\Theta j}$. The number, $n_{\Theta j}$, is a natural number rounded off to the nearest whole number. Coordinates in the R_{jk} - Θ_{jk} - Z_{jk} coordinate system are denoted as $(r_{Fjk}, \theta_{Fjk}, z_{Fjk})$.

The total number of elliptical mirrors is denoted as n_M . Note that when j and k are 0, it is a single elliptical mirror. A multi-part mirror disperses the first focal points over its surface more widely than a single elliptical mirror. Using a multi-part mirror, it is possible to concentrate diffused light from a wider area than using a single elliptical mirror. In the designs of the single elliptical and multi-part mirrors, the number of mirrors n_M , angle ξ_{Fj} , number $n_{\Theta j}$, angle θ_{Fj} , coordinate r_{Fjk} , and coordinate z_{Fjk} are determined using the equations below. Using Eq. (1), coordinates $(r_{F00}, \theta_{F00}, z_{F00})$ can be expressed as

$$\begin{pmatrix} r_{F00} \\ \theta_{F00} \\ z_{F00} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -2f \end{pmatrix} \quad (5)$$

Using Eqs. (1) and (3), the relationship between angle ξ_F , focal length f , radius r_E , and distance g can be expressed as

$$\tan \frac{\xi_F}{2} = \frac{r_E + \frac{g}{2}}{2f} \quad (6)$$

Using Eq. (6), angle ξ_{Fj} can be expressed as

$$\xi_{Fj} = j\xi_F \quad (j = 0, 1, \dots, n_R - 1) \quad (7)$$

Using Eqs. (5) and (7), coordinates $(r_{Fj0}, \theta_{Fj0}, z_{Fj0})$ can be expressed as

$$\begin{pmatrix} r_{Fj0} \\ \theta_{Fj0} \\ z_{Fj0} \end{pmatrix} = \begin{pmatrix} \cos \xi_{Fj} & 0 & -\sin \xi_{Fj} \\ 0 & 1 & 0 \\ \sin \xi_{Fj} & 0 & \cos \xi_{Fj} \end{pmatrix} \begin{pmatrix} r_{F00} \\ \theta_{F00} \\ z_{F00} \end{pmatrix} \quad (8)$$

Using Eqs. (3) and (8), the relationship between angle θ_{Fj} , radius r_E , and radius r_{Fj0} can be expressed as

$$\sin \frac{\theta_{Fj}}{2} = \frac{r_E}{r_{Fj0}} \quad (9)$$

Using Eq. (9), number $n_{\Theta j}$ can be expressed as

$$n_{\Theta j} = \left\lceil \frac{2\pi}{\theta_{Fj}} \right\rceil \quad (10)$$

Using Eq. (10), angle θ_{Fj} can be expressed as

$$\theta_{Fj} = \frac{2\pi}{n_{\Theta j}} \quad (11)$$

Using Eqs. (10) and (11), angle θ_{Fjk} can be expressed as

$$\theta_{Fjk} = k\theta_{Fj} \quad (k = 0, 1, \dots, n_{\Theta j} - 1) \quad (12)$$

Using Eqs. (8) and (12), coordinates $(r_{Fjk}, \theta_{Fjk}, z_{Fjk})$ can be expressed as

$$\begin{pmatrix} r_{Fjk} \\ \theta_{Fjk} \\ z_{Fjk} \end{pmatrix} = \begin{pmatrix} 0 \\ \theta_{Fjk} \\ 0 \end{pmatrix} + \begin{pmatrix} r_{Fj0} \\ \theta_{Fj0} \\ z_{Fj0} \end{pmatrix} \quad (13)$$

Using Eq. (10), the number of mirrors, n_M , can be expressed as

$$n_M = \sum_{j=0}^{n_R-1} n_{\Theta j} \quad (14)$$

To evaluate the light-concentrating performance of a single elliptical mirror, we designed three types of it with different focal lengths f and radii r_E . Table 2 lists the design values for these single elliptical mirrors. The design values of the single elliptical mirror with a focal length f of 199.929 mm and radius r_E of 1 mm are based on the values provided by Matsumoto [19]. To evaluate the light-concentrating performance of the multi-part mirrors, we designed two types with different focal lengths f . The design values of the multi-part mirrors are summarized in Tables 3 and 4. Table 3 and 4 list the data for multi-part mirrors with focal lengths f of 17.503 mm and 199.929 mm, respectively. The design values in Table 4 are based on the values obtained by Matsumoto [19]. In Tables 2-4, the design values are determined using Eqs. (1), (3), (4), (14), (7), (10), (11), and (13) for focal length f , radius r_E , height h , number of mirrors n_M , angle ξ_{Fj} , number $n_{\Theta j}$, angle θ_{Fj} , and coordinates r_{Fjk} and z_{Fjk} , respectively.

3. Analysis Method

In single elliptical and multi-part mirrors, the diffused light incident on the elliptical mirrors is reflected by the mirrored surface and concentrated on the collector surface at the second focal point. To evaluate the light-concentrating performance of the designed single elliptical and multi-part mirrors, we analyzed how much of the light incident on them was concentrated on their collector surface. Figure 3 shows the analysis method for these single elliptical and multi-part

Table 2: Design values of length of major axis, length of minor axis, focal length, radius of elliptical mirror, height, number of elliptical mirrors in R-axis, number of all elliptical mirrors, angle from R_{00} - Θ_{00} - Z_{00} to R_{00} - Θ_{00} - Z_{00} , number of elliptical mirrors in Θ -axis, angle from F_{00} to F_{01} , and coordinate of F_{00} in single elliptical mirrors.

l [mm]	s [mm]	f [mm]	r_E [mm]	h [mm]	n_R	n_M	ξ_{F0} [°]	$n_{\Theta 0}$	θ_{F0} [°]	r_{F00} [mm]	z_{F00} [mm]
18.000	4.200	17.503	0.980	0.497	1	1	0.000	1	0.000	0.000	-35.006
40.300	36.300	17.504	32.697	22.796	1	1	0.000	1	0.000	0.000	-35.008
200.000	14.100	199.929	1.000	0.498	1	1	0.000	1	0.000	0.000	-399.858

Table 3: Design values of length of major axis, length of minor axis, focal length, radius of elliptical mirror, height, distance from one edge to another edge of mirror surface, number of elliptical mirrors in R-axis, number of all elliptical mirrors, number indicating elliptical mirror in R-axis, angle from R_{00} - Θ_{00} - Z_{00} to R_{j0} - Θ_{j0} - Z_{j0} , number of elliptical mirrors in Θ -axis, angle from F_{j0} to F_{j1} , and coordinates of F_{jk} in multi-part mirror when f is 17.503 mm.

		l [mm]	s [mm]	f [mm]	r_E [mm]	h [mm]	g [mm]	n_R	n_M		
		18.000	4.200	17.503	0.980	0.497	0.200	17	810		
j	ξ_{Fj} [°]	$n_{\Theta j}$	θ_{Fj} [°]	r_{Fjk} [mm]	z_{Fjk} [mm]	j	ξ_{Fj} [°]	$n_{\Theta j}$	θ_{Fj} [°]	r_{Fjk} [mm]	z_{Fjk} [mm]
0	0.000	1	0.000	0.000	-35.006	9	33.608	56	6.429	19.376	-29.154
1	3.734	6	60.000	2.280	-34.932	10	37.342	61	5.902	21.234	-27.831
2	7.468	13	27.692	4.550	-34.709	11	41.077	66	5.455	23.001	-26.389
3	11.203	19	18.947	6.801	-34.339	12	44.811	71	5.070	24.671	-24.835
4	14.937	26	13.846	9.023	-33.823	13	48.545	76	4.737	26.236	-23.175
5	18.671	32	11.250	11.207	-33.164	14	52.279	80	4.500	27.690	-21.417
6	22.405	38	9.474	13.343	-32.363	15	56.014	84	4.286	29.026	-19.568
7	26.140	44	8.182	15.422	-31.426	16	59.748	87	5.294	30.239	-17.636
8	29.874	50	7.200	17.436	-30.355	-	-	-	-	-	-

mirrors. An elliptical mirror is three-dimensional in shape, which is obtained by rotating an ellipse around the Z_{jk} axis on the R_{jk} - Z_{jk} plane of the R_{jk} - Θ_{jk} - Z_{jk} coordinate system. Therefore, to achieve axisymmetry with respect to the Z_{jk} axis, each elliptical mirror is analyzed in the R_{jk} - Z_{jk} plane in the R_{jk} - Θ_{jk} - Z_{jk} coordinate system. We analyzed the designed single elliptical and multi-part mirrors in the R-Z plane in the R- Θ -Z coordinate system.

The number of reflections of the light in the mirror surface is denoted as m , where m is a natural number including 0. When m is 0, it is assumed that the light enters the elliptical mirror. This occurs from a line perpendicular to the Z_{jk} axis that passes through the first focal point, F_{jk} , and is parallel to the R_{jk} axis. The point of incidence R_0 is defined as the point where the parallel line intersects with the incident light. The coordinates of the point of incidence, R_0 , are $(r_{R0}, -2f)$. Note that the absolute value of the coordinate, r_{R0} , is less than or equal to the radius, r_E . The angle between the point of incidence of the light, R_0 , and the R_{jk} axis is denoted as ϕ_{R0} . The reflection points on the mirror surface are denoted as R_{m-1} and R_m . The coordinates of reflection point R_{m-1} are (r_{Rm-1}, z_{Rm-1}) . The angle between the normal of the ellipse and the R_{jk} axis at the point of reflection, R_{m-1} , is denoted as ϕ_{Nm-1} . The angle between the light reflected at the point of reflection, R_{m-1} , and the R_{jk} axis is denoted as ϕ_{Rm-1} . The coordinates of the reflection point R_m are (r_{Rm}, z_{Rm}) . The angle between the normal of the ellipse and the R_{jk} axis at the point of reflection, R_m , is denoted as ϕ_{Nm} . The angle between the light reflected at the point of reflection, R_m , and the R_{jk} axis is denoted as ϕ_{Rm} .

The collector surface is a line on the R axis, perpendicular to the Z axis, and passing through the origin, O, which is the second focal point. The R-Z plane is a plane obtained by rotating the R_{00} - Z_{00} plane about the origin O_{00} by an angle ξ_{Fj} . The point of concentration, C, is defined as the point of intersection of the light rays on the collector surface at coordinates $(r_C, 0)$.

In the analysis of the designed single elliptical and multi-part mirrors, the relationship between the angle, ϕ_{R0} , of the coordinate, r_{R0} , of the light incident on the elliptical mirrors and the coordinate, r_C , of the surface where the incoming light collects is found using the equations below. The equation describing the light passing through the point of reflection R_{m-1} in the R_{jk} - Z_{jk} plane can be expressed as follows, where a and b are variables.

$$z_{jk} = ar_{jk} + b \quad (15)$$

Variable a can be expressed as

$$a = \tan \phi_{Rm-1} \quad (16)$$

Variable b can be expressed as

$$b = z_{Rm-1} - \tan \phi_{Rm-1} r_{Rm-1} \quad (17)$$

Using Eqs. (2) and (15), coordinate r_{Rm} can be expressed as

$$r_{Rm} = \frac{-as^2(b+2f)}{l^2 + a^2s^2} + \frac{s\sqrt{a^2s^2(b+2f)^2 - (l^2 + a^2s^2)\{(b+2f)^2 - l^2\}}}{l^2 + a^2s^2} \quad (18)$$

Table 4: Design values of length of major axis, length of minor axis, focal length, radius of elliptical mirror, height, distance from one edge to another edge of mirror surface, number of elliptical mirrors in R-axis, number of all elliptical mirrors, number indicating elliptical mirror in R-axis, angle from $R_{00}-\Theta_{00}-Z_{00}$ to $R_{j0}-\Theta_{j0}-Z_{j0}$, number of elliptical mirrors in Θ -axis, angle from F_{j0} to F_{j1} , and coordinates of F_{jk} in multi-part mirror when f is 199.929 mm.

		l [mm]		s [mm]	f [mm]	r_E [mm]	h [mm]	g [mm]	n_R	n_M	
		200.000		14.100	199.929	1.000	0.498	1.500	12	410	
j	ξ_{Fj} [°]	$n_{\theta j}$	θ_{Fj} [°]	r_{Fjk} [mm]	z_{Fjk} [mm]	j	ξ_{Fj} [°]	$n_{\theta j}$	θ_{Fj} [°]	r_{Fjk} [mm]	z_{Fjk} [mm]
0	0.000	1	0.000	0.000	-399.005	6	3.015	37	9.728	20.990	-398.452
1	0.502	6	59.988	3.500	-398.989	7	3.518	43	8.370	24.484	-398.253
2	1.005	12	29.994	7.000	-398.943	8	4.020	50	7.196	27.977	-398.023
3	1.507	18	19.996	10.499	-398.867	9	4.523	56	6.428	31.467	-397.762
4	2.010	25	14.398	13.997	-398.759	10	5.025	62	5.804	34.955	-397.471
5	2.512	31	11.608	17.494	-398.621	11	5.528	69	5.217	38.440	-397.149

Figure 3: Analysis method of single elliptical and multi-part mirrors.

Using Eqs. (15) and (18), coordinate z_{Rm} can be expressed as

$$z_{Rm} = ar_{Rm} + b \quad (19)$$

Using Eqs. (18) and (19), angle ϕ_{Nm} can be expressed as

$$\tan \phi_{Nm} = \frac{s^2 z_{Rm}}{l^2 r_{Rm}} \quad (20)$$

Using Eq. (20), angle ϕ_{Rm} can be expressed as

$$\phi_{Rm} = 2\phi_{Nm} - \phi_{Rm-1} \quad (21)$$

In the equation describing the light passing through the point of reflection, R_m , using Eq. (15), variable a can be expressed as

$$a = \tan \phi_{Rm} \quad (22)$$

Variable b can be expressed as

$$b = z_{Rm} - \tan \phi_{Rm} r_{Rm} \quad (23)$$

Using Eqs. (15), (22), and (23), the equation describing the light passing through the point of reflection, R_m , in the R_{jk} - Z_{jk} plane can be expressed as indicated below. Using Eq. (7), the coordinates (r, z) can be expressed as

$$\begin{pmatrix} r \\ z \end{pmatrix} = \begin{pmatrix} \cos \xi_{Fj} & -\sin \xi_{Fj} \\ \sin \xi_{Fj} & \cos \xi_{Fj} \end{pmatrix} \begin{pmatrix} r_{00} \\ z_{00} \end{pmatrix} \quad (24)$$

Using Eqs. (7), (22), and (23), the equation describing the light passing through the point of reflection, R_m , in the R - Z plane can be expressed as

$$z = \frac{\sin \xi_{Fj} + a \cos \xi_{Fj}}{\cos \xi_{Fj} - a \sin \xi_{Fj}} r + \frac{b}{\cos \xi_{Fj} - a \sin \xi_{Fj}} \quad (25)$$

Using Eq. (25), the coordinate, r_C , can be expressed as

$$r_C = -\frac{b}{\sin \xi_{Fj} + a \cos \xi_{Fj}} \quad (26)$$

4. Experimental Method

4.1 Manufacturing Mirrors Using the design values listed in Table 2, we manufactured actual single elliptical mirrors with a focal length f of 17.503 mm and a radius r_E of 0.98 mm and with a focal length f of 17.504 mm and a radius r_E of 32.697 mm. Fig. 4 shows images of the single elliptical mirror. Fig. 4(a) shows a single elliptical mirror with a focal length f of 17.503 mm and a radius r_E of 0.98 mm, and Fig. 4(b) shows a single elliptical mirror with a focal length f of 17.504 mm and a radius r_E of 32.697 mm. For the single elliptical mirror with a focal length f of 17.503 mm and a radius r_E of 0.98 mm, the substrate is black polycarbonate. Aluminum was vapor-deposited on

Table 5: Specifications of light receiver, light-shielding sheet, first focal point, and second focal point in measured objects of dome light, light-shielded single elliptical mirror, light-shielded multi-part mirror, single elliptical mirror, and multi-part mirror in experimental set-up.

Object name	Light receiver	Light-shielding sheet	First focal point	Second focal point
Dome light	F	-	-	-
Light-shielded single elliptical mirror	O	F	F	O
Light-shielded multi-part mirror	O	S	F	O
Single elliptical and multi-part mirrors	O	-	F	O

(a) f is 17.503 mm, and r_E is 0.98 mm

(b) f is 17.504 mm, and r_E is 32.697 mm

Figure 4: Images showing elliptical mirror and light-shielding sheet in single elliptical mirrors.

Figure 5: Image showing elliptical mirror, light-shielding sheet, and base in multi-part mirror when f is 17.503 mm.

the mirror surface, after which silicon dioxide was vapor-deposited as a protective film. The arithmetic mean roughness of the mirror surface is $1.17 \mu\text{m}$. For the single elliptical mirror with a focal length f of 17.504 mm and a radius r_E of 32.697 mm, the substrate is heat-resistant glass. Aluminum was vapor-deposited on the mirror surface, after which silicon dioxide was vapor-deposited as a protective film. The reflectivity of the mirror surface is more than 88% at wavelengths from 400 nm to 700 nm. The parts other than the mirror surface of this single elliptical mirror were covered with a light-shielding sheet.

Using the design values listed in Table 3, we manufactured an actual multi-part mirror with a focal length f of 17.503 mm, whose images are displayed in Fig. 5. The

Figure 6: Mechanism of white LED, diffuser, and light-shielding sheet in dome light; measured object; and light receiver in experimental set-up from (a) plan, (b) longitudinal, and (c) front views.

multi-part mirror is composed of multiple elliptical mirrors and a base. The elliptical mirrors of the multi-part mirror are identical to the manufactured single elliptical mirror with a focal length f of 17.503 mm and a radius r_E of 0.98 mm. The base of the multi-part mirror is made of an aluminum alloy. The surface of the base was anodized in matte black to prevent light from reflecting off it and subsequently covered with a light-shielding sheet.

4.2 Experimental Set-up Figure 6 shows the structure of the experimental set-up. Figures 6(a)-(c) present the plan, longitudinal, and front views, respectively. The experimental set-up consists of a dome light, measured object, and light receiver. The experimental set-up is covered with a light-shielding sheet to block stray light that could affect

(a) Plan view

(b) Side view

(c) Front view

Figure 7: Images of single elliptical mirror when f is 17.503 mm and r_E is 0.98 mm, light-shielding sheet, illuminance meter, power source, dome light, and light receiver in the experimental set-up from (a) plan, (b) side, and (c) front views.

Figure 8: Image of light diffused by diffuser and light-shielding sheet in dome light.

the measurements. An LED light source and a diffuser are installed inside the dome light. The diffuser is in the shape of a half celestial sphere created by rotating a semicircle on the R-Z plane in the R- Θ -Z coordinate system around the Z axis. The equation of the semicircle can be expressed as

$$r^2 + (z + 2f)^2 = 40000 \quad (z \geq -2f) \quad (27)$$

The diffuser can generate diffused light inside the dome light. The measured object and the light receiver are positioned with the Z axis as the central axis. Depending on what is to be measured, the measured object and the light receiver are fixed at positions along the Z axis.

Table 5 lists the specifications of the measured objects in the experimental set-up. Figure 6 shows the positions of the

light-shielding sheet, measured object, and light receiver in the measured object in the experimental set-up. The light receivers are installed at the focal point, F, and the origin, O, and oriented in the positive and negative direction of the Z axis, respectively. The light-shielding sheet is installed at the focal point, F, for the light-shielded single elliptical mirror and at point S for the light-shielded multi-part mirror. For the light-shielded single elliptical and multi-part mirrors and the single elliptical and multi-part mirrors, the first focal point is set at the focal point, F, and the second focal point is set at the origin, O.

Figure 7 shows the images of the experimental set-up, with Figs. 7(a)-(c) displaying the plan, side, and front views, respectively. In Fig. 7, the measured object is the single elliptical mirror with a focal length f of 17.503 mm and a radius r_E of 0.98 mm. The dome light is connected to a power source. The light receiver is connected to an illuminance meter. The dome light is model HPD2-400SW-CR14, and the power source is model PD2-5024, both manufactured by CCS Inc. HPD2-400SW-CR14 can generate diffused light by illuminating the diffuser inside the dome light from the white LED installed inside the dome light. PD2-5024 allows a dimmer control of the dome light. A total of 16 levels of coarse dimming and 16 levels of fine dimming can be combined to produce 256 levels of dimming. In the experimental set-up, we used coarse dimming to adjust the dimming level and fixed the fine dimming to one level. The light receiver and illuminance meter are model T-10MA from Konica Minolta Inc. T-10MA can measure the illuminance at the light receiver and display the measured illuminance on the illuminance meter. The light receiver has a radius of 7 mm, and the light-receiving direction is hemispherical. The range of illuminance that can be measured is from 0.01 lx to 299,900 lx.

Figure 8 shows an image of the diffused light on the inside of the dome light. The dimming level of PD2-5024 was set as 2. The image was captured with Kodak SP360. SP360 can shoot at a Θ -axis direction of 360° and a Z-axis direction of 214° . The lens of SP360 was placed at the focal point, F, and oriented in the positive Z-axis direction. The figure shows the diffused light generated from the diffuser at 360° in the Θ direction and 180° in the Z direction, except at the light receiver.

4.3 Measurement The illuminance of the dome light, light-shielded measured object, measured object, and measured object only and the illuminance ratio indicating the light-concentrating efficiency are denoted as i_D , i_S , i_M , i_T , and e , respectively. Illuminance i_T is the value obtained by subtracting illuminance i_S from illuminance i_M . The illuminance ratio, e , is the ratio of illuminance i_T to illuminance i_D . Illuminance i_T can be expressed as

$$i_T = i_M - i_S \quad (28)$$

Using Eq. (28), the illuminance ratio, e , can be expressed as

$$e = \frac{i_T}{i_D} \quad (29)$$

Table 6: Comparison of dimming level with illuminances of dome light, light-shielded measurement object, measured object, and only measured object as well as the illuminance ratio of single elliptical mirrors.

f [mm]	r_E [mm]	n_M	Dimming level	i_D [lx]	i_S [lx]	i_M [lx]	i_T [lx]	e
17.503	0.980	1	2	1,031.0	241.3	241.6	0.3	0.0002910
17.504	32.697	1	2	1,031.0	241.3	538.0	296.7	0.2870000
199.929	1.000	1	2	1,172.0	-	-	-	0.0000479

We experimentally determined the relationship between the dimming level and illuminance i_D of the dome light. We measured illuminance i_D with the dimming level of the dome light set as 2. We compared the measured illuminance i_S and i_M of the measured object to each other and to illuminance i_T of the measured object only. Subsequently, we determined illuminance i_T from illuminance i_S and i_M . We compared the illuminance ratio, e , in the measured object and then determined it from illuminance i_T .

5. Performance Evaluation

5.1 Analysis Results We analyzed the relationship between angle ϕ_{R0} and coordinate r_C of the single elliptical and multi-part mirrors. We used Eqs. (15)-(26) for this by calculating the value of coordinate r_C at 1° increments of angle ϕ_{R0} . The range of angle ϕ_{R0} was set from -90° to 90° to cover a half celestial sphere of 180° in the Z-axis direction. Coordinate r_C ranged from -7 mm to 7 mm to cover the radius of the light receiver. We compared the single elliptical mirrors with a focal length f of 17.503 mm and a radius r_E of 0.98 mm and with a focal length f of 199.929 mm and a radius r_E of 1 mm. We also compared the multi-part mirrors with focal lengths f of 17.503 mm and 199.929 mm. The ratio of coordinate r_{R0} to radius r_E is denoted as r_{R0}/r_E . For the single elliptical and multi-part mirrors, we compared the relationship between angle ϕ_{R0} and coordinate r_C symmetrically about the Z axis at the following values of r_{R0}/r_E : 0, 0.0979, 0.5, and 1. For the multi-part mirrors, we compared the relationship between angle ϕ_{R0} and coordinate r_C symmetrically about the Z axis at the following values of r_{R0}/r_E : -1, -0.5, -0.0979, 0, 0.0979, 0.5 and 1. In each case, we took measurements at angles ξ_{F0} , ξ_{F8} , and ξ_{F16} or alternatively at angles ξ_{F0} and ξ_{F11} . We plotted all the relationships between angle ϕ_{R0} and coordinate r_C at r_{R0}/r_E values of -0.0979 and 0.0979 for the single elliptical mirror with a focal length f of 17.503 mm and a radius r_E of 0.98 mm and the multi-part mirror with a focal length f of 17.503 mm at angle ξ_{F0} .

Figures 9-12 compare the relationships for the single elliptical mirrors, and Figs. 13-16 compare them for the multi-part mirror with a focal length f of 17.503 mm at angles ξ_{F8} , and ξ_{F16} . Figures 17-20 compare the relationships for the multi-part mirror with focal lengths f of 17.503 mm and 199.929 mm. In these figures when the ratio of r_{R0}/r_E is greater than the range from 0 to 1 or smaller than the range from 0 to -1, less of the light incident from -90° to 90° , which is the range of angle ϕ_{R0} , is concentrated from -7 mm to 7 mm, which is the range of coordinate r_C . When the focal length, f , and the radius, r_E , of the single elliptical mirrors are increased, less light is concentrated at each

Figure 9: Comparison of relationship of angle between incident light at R_0 and R_{jk} and coordinate of C when r_{R0}/r_E is 0 in single elliptical mirrors.

Figure 10: Comparison of relationship of angle between incident light at R_0 and R_{jk} and coordinate of C when r_{R0}/r_E is 0.0979 in single elliptical mirrors.

Figure 11: Comparison of relationship of angle between incident light at R_0 and R_{jk} and coordinate of C when r_{R0}/r_E is 0.5 in single elliptical mirrors.

ratio r_{R0}/r_E . In the case of the multi-part mirrors, less light is concentrated as the angle, ξ_{Fj} , and the focal length, f , increase.

5.2 Experiment Results Figure 21 shows the appearance of the measured objects inside the experimental set-up during the experiment. Figures 21(a)-(c) present the dome light, light-shielded single elliptical mirrors with a focal length f of 17.503 mm and a radius r_E of 0.98 mm and with a focal length f of 17.504 mm and a radius r_E of 32.697 mm, and (c) light-shielded multi-part mirror with a focal

Figure 12: Comparison of relationship of angle between incident light at R_0 and R_{jk} and coordinate of C when r_{R0}/r_E is 1 in single elliptical mirrors.

Figure 13: Comparison of relationship of angle between incident light at R_0 and R_{jk} and coordinate of C when r_{R0}/r_E is 0 at ξ_{F0} , ξ_{F8} , and ξ_{F16} in multi-part mirror when f is 17.503 mm.

Figure 14: Comparison of relationship of angle between incident light at R_0 and R_{jk} and coordinate of C when r_{R0}/r_E is -0.0979 and 0.0979 at ξ_{F0} , ξ_{F8} , and ξ_{F16} in multi-part mirror when f is 17.503 mm.

length f of 17.503 mm, respectively. Figures 21(d)-(f) show the single elliptical mirror with a focal length f of 17.503 mm and radius r_E of 0.98 mm, single elliptical mirror with a focal length f of 17.504 mm and radius r_E of 32.697 mm, and multi-part mirror with a focal length f of 17.503 mm, respectively. This is the view from the negative Z-axis direction with the dimming level set as 2. The images of the dome light and the light-shielded single elliptical and multi-part mirrors demonstrate the light-shielding sheet reflecting the diffused light. Therefore, it is necessary to determine illuminance i_T , which is the difference between illuminance i_M and i_S . The images of the single elliptical and multi-part mirrors show that the mirrors can brightly reflect the diffused light.

Table 6 lists the dome light dimming level, illuminance i_D , i_S , i_M , and i_T values, and illuminance ratio e for the sin-

Figure 15: Comparison of relationship of angle between incident light at R_0 and R_{jk} and coordinate of C when r_{R0}/r_E is -0.5 and 0.5 at ξ_{F0} , ξ_{F8} , and ξ_{F16} in multi-part mirror when f is 17.503 mm.

Figure 16: Comparison of relationship of angle between incident light at R_0 and R_{jk} and coordinate of C when r_{R0}/r_E is -1 and 1 at ξ_{F0} , ξ_{F8} , and ξ_{F16} in multi-part mirror when f is 17.503 mm.

Figure 17: Comparison of relationship of angle between incident light at R_0 and R_{jk} and coordinate of C when r_{R0}/r_E is 0 at ξ_{F0} and ξ_{F11} in multi-part mirrors.

gle elliptical mirrors. Illuminance i_M is greater than illuminance i_S . Therefore, it is necessary to obtain illuminance i_T , which is the difference between illuminance i_M and illuminance i_S . The illuminance ratio, e , for the single elliptical mirror with a focal length f of 199.929 mm and radius r_E of 1 mm was estimated based on previous results of Matsumoto [19]. The relationship between the number of mirrors, n_M , and the illuminance ratio, e , can be expressed as

$$e = -0.000000004741n_M^2 + 0.00001138n_M + 0.00003655 \quad (30)$$

Figure 18: Comparison of relationship of angle between incident light at R_0 and R_{jk} and coordinate of C when r_{R0}/r_E is -0.0979 and 0.0979 at ξ_{F0} and ξ_{F11} in multi-part mirrors.

Figure 19: Comparison of relationship of angle between incident light at R_0 and R_{jk} and coordinate of C when r_{R0}/r_E is -0.5 and 0.5 at ξ_{F0} and ξ_{F11} in multi-part mirrors.

Figure 20: Comparison of relationship of angle between incident light at R_0 and R_{jk} and coordinate of C when r_{R0}/r_E is -1 and 1 at ξ_{F0} and ξ_{F11} in multi-part mirrors.

Using Eq. (30), the illuminance ratio, e , when n_M is 1 is estimated as 0.0000479. Because illuminance i_D is different for the three single elliptical mirrors, we compared their values of the illuminance ratio, e . First, we compared the single elliptical mirror with a focal length f of 17.503 mm and a radius r_E of 0.98 mm to that with a focal length f of 17.504 mm and a radius r_E of 32.697 mm. As the radius, r_E , increases, the illuminance ratio, e , also increases. When the radius, r_E , is 33.3 times as large, the illuminance ratio, e , is 986 times larger. Next, we compared the single elliptical

Figure 21: Images of measured objects with dome light, light-shielded single elliptical and multi-part mirrors, single elliptical and multi-part mirrors in experimental set-up.

Figure 22: Comparison of relationship between number of all elliptical mirrors and illuminance ratio in multi-part mirrors.

mirror with a focal length f of 17.503 mm and a radius r_E of 0.98 mm to that with a focal length f of 199.929 mm and a radius r_E of 1 mm. As the focal length f decreases, the illuminance ratio, e , increases. When the focal length f is 0.0875 times as large, the illuminance ratio, e , is 6.07 times larger. This shows that to increase the illuminance ratio, e , it is more effective to change the radius, r_E , than modify the focal length, f .

Figure 22 compares the relationship between the number of mirrors, n_M , and the illuminance ratio, e , for the multi-part mirrors. We also compared the multi-part mirrors with focal lengths f of 17.503 mm and 199.929 mm. For the multi-part mirror with a focal length f of 199.929 mm, the relationship between the number of mirrors, n_M , and the illuminance ratio, e , is the result obtained by Matsumoto [19]. As the number of mirrors, n_M , increases, the illuminance ratio, e , increases. The rate of increase in the illuminance ratio, e , is greater for the multi-part mirror with a focal length f of 17.503 mm than that for the multi-part mirror with a focal length f of 199.929 mm. Furthermore, the increase in the illuminance ratio, e , is nonlinear. As the number of mirrors, n_M , increases, the rate of increase in the illuminance ratio, e , decreases. The analysis results in Figs. 13-20 show that as the angle, ξ_{Fj} , and the focal length, f , increase, less light is concentrated. To solve this problem, it is necessary for the multi-part mirror to possess a three-dimensional collector instead of a flat collector surface to accommodate a large angle ξ_{Fj} .

6. Conclusion

In this study, we installed a collector surface on the second focal point of an elliptical mirror to increase its light-concentrating efficiency. Subsequently we evaluated single elliptical and multi-part mirrors that capture diffused light from a half celestial sphere not incident on the first focal point and concentrate it on the collector surface as well as the second focal point. The results obtained in this study are as follows:

1. We configured a coordinate system to design single elliptical and multi-part mirrors and subsequently analyzed the mechanism by which the light would be incident on the designed single elliptical and multi-part mirrors and concentrate on their collector surfaces. Based on the design values, we manufactured actual single elliptical and multi-part mirrors, which were illuminated using our experimental set-up with diffused light and measured the illuminance of the light concentrated on their collector surfaces.
2. We analyzed how the light concentrated on the collector surfaces of the actual single elliptical and multi-part mirrors. As we increased the focal length and the radius of the single elliptical mirrors, we found less light concentrated at each ratio of the coordinate of the point of incidence to the radius. In the case of the multi-part mirrors, less light was concentrated as we increased the angles and the focal lengths of the elliptical mirrors.
3. We compared the illuminance ratio, which is a measure of the light-concentrating performance, of the single elliptical and multi-part mirrors. For the single elliptical mirrors, as the radius of the elliptical mirror increases, the illuminance ratio also increases. When the radius is 33.3 times as large, the illuminance ratio is 986 times larger. As the focal length of the elliptical mirror decreases, the illuminance ratio increases.

When the focal length is 0.0875 times as large, the illuminance ratio is 6.07 times larger. Therefore, to increase the illuminance ratio, it is more effective to change the radius instead of the focal length. For the multi-part mirrors, we found that increasing the number of elliptical mirrors increases the illuminance ratio. As the focal length decreases, the illuminance ratio increases. As the number of mirrors increases, the illuminance ratio increases. The results of our analysis suggest that as the number of mirrors increases, the angles of the elliptical mirrors become larger, thereby decreasing the amount of concentrated light and slowing the rate of increase in the illuminance ratio. To solve this problem, it is necessary to install a three-dimensional collector in a multi-part mirror instead of a flat collector surface to accommodate a larger angle.

Acknowledgements

This study was funded by the Kurata Grant awarded by the Hitachi Global Foundation, Yashima Environment Technology Foundation, and Takahashi Industrial and Economic Research Foundation. The author expresses his appreciation for the same.

References

- [1] Japan Solar Energy Society ed., "New Solar Energy Utilization Handbook", *Japan Solar Energy Society*, pp.123-128, Tokyo, 2015 (in Japanese).
- [2] Japan Solar Energy Society ed., "Solar Energy Utilization Technology", *Japan Solar Energy Society*, p.66, Tokyo, 2006 (in Japanese).
- [3] Geospatial Information Authority of Japan ed., "National Atlas of Japan", *Japan Map Center*, p.47, Tokyo, 1990 (in Japanese).
- [4] R. Winston, "Principles of Solar Concentrators of a Novel Design", *Solar Energy*, Vol.16, No.2, pp.89-95, 1974.
- [5] G. H. Derrick, I. M. Bassett and D. R. Mills, "Comparison of Reflector Designs for Stationary Tubular Solar Collectors", *Solar Energy*, Vol.37, No.3, pp.195-203, 1986.
- [6] S. S. Rao, H. Lee and Y. Hu, "Optimal Design of Compound Parabolic Concentrator Solar Collector System", *Transactions of the ASME, Journal of Mechanical Design*, Vol.136, No.9, pp.091402-1-091402-10, 2014.
- [7] G. R. Mather and D. C. Beekley, "Performance of an Evacuated Tubular Collector Using Non-imaging Reflectors", *Proceedings of the Joint Conference International Solar Energy Society*, No.2, pp.64-78, 1976.
- [8] D. R. Mills and J. E. Giutronich, "Symmetrical and Asymmetrical Ideal Cylindrical Radiation Transformers and Concentrators", *Journal of the Optical Society of America*, Vol.69, No.2, pp.325-328, 1979.
- [9] I. M. Bassett and G. H. Derrick, "Diffuse Reflectors in Non-imaging Optics", *Proceedings of the International Solar Energy Society Silver Jubilee Congress*, Vol.1, pp.557-560, 1979.

- [10] D. P. Grimmer, "A Comparison of Compound Parabolic and Simple Parabolic Concentrating Solar Collectors", *Solar Energy*, Vol.22, No.1, pp.21-25, 1979.
- [11] J. R. Frissora and D. M. Platt, "Drainable Evacuated Collector with Compound Parabolic Cusp Reflector", *Solar Energy and Conservation*, Vol.1, pp.113-140, 1980.
- [12] B. Norton, P. C. Eames and Y. P. Yadav, "Symmetric and Asymmetric Linear Compound Parabolic Concentrating Solar Energy Collectors - The State-of-the-art in Optical and Thermo-physical Analysis -", *International Journal of Ambient Energy*, Vol.12, No.4, pp.171-190, 1991.
- [13] P. C. Eames and B. Norton, "The Effect of Sky Conditions on the Partition of Incident Solar Energy between the Components of a CPC Solar Energy Collector", *Proceedings of the Solar World Congress 1991 (SWC1991)*, Vol.2, pp.1884-1889, 1992.
- [14] A. Suzuki, "Optimum Design for Non-imaging Solar Concentrators with Reference to Scattering Insolation Fraction", *Proceedings of the Japan Solar Energy Society Japan Wind Energy Association Joint Conference*, pp.37-40, 1993 (in Japanese).
- [15] P. C. Eames, B. Norton, Y. Tripagnostopoulos and P. Yianoulis, "Modelling Line-axis Solar Concentrators in the Medium Temperature Range", *Renewable Energy*, Vol.16, No.1, pp.743-748, 1999.
- [16] J. Wang, L. Yu, C. Jiang, S. Yang and T. Liu, "Optical Analysis of Solar Collector with New V-shaped CPC", *Solar Energy*, Vol.135, pp.780-785, 2016.
- [17] N. Yamada and Y. Honda, "Optical Analysis of High Concentrating Optical Geometry Coupled with Diffuse Solar Radiation Absorber", *Journal of Japan Solar Energy Society*, Vol.36, No.3, pp.45-50, 2010 (in Japanese).
- [18] N. Sellami, T. K. Mallick and D. A. Mcneil, "Optical Characterisation of 3-D Static Solar Concentrator", *Energy Conversion and Management*, Vol.64, pp.579-586, 2012.
- [19] M. Matsumoto, "Multi-part Mirror for Focusing Diffuse Light", *Mechanical Engineering Journal*, Vol.4, No.3, 2017, DOI:10.1299/mej.16-00520.

M. Matsumoto (Member) received his Ph.D. degree from University of Tsukuba. He is currently an associate professor at Kanagawa University. His current research interest is mechanical engineering. He is a member of IIAE and so on.