The Orthogonal Transform Sign

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Abstract

The nature of transform signs in typical real orthogonal transforms are presented. Phase-only synthesis is introduced by "Image recovery -Theory and Application-" written by Henry Stark. Sign-only synthesis is the extended version of the Phase-only synthesis. Sign-only synthesis is defined as an inverse transform of only signs of the transform coefficients and has interesting property in its applications. Orthogonal transforms taken in this paper are Discrete Cosine Transform, Hadamard transform, Fourier Transform with binarized phase and Karhunen-Loeve Transform. These transforms have real number coefficients and then it looks interesting to know the property of each transform signs. The characteristics of these Sign-only synthesis are compared each other. And interesting nature of transform signs are extended. One application of Sign-only synthesis is shown as a facial authentication.

Keywords: Orthogonal transforms, Sign-only synthesis, Discrete Cosine Transform, Hadamard Transform, Fourier Transform with binarized phase, Karhunen-Loeve Transform.

1 Introduction

In signal analysis there are mainly two ways: One is in the time domain or the original space domain and the other is in the frequency domain. Each analysis has its own merit. In time domain we can see directly whether the signal has quick changing parts or not but it is difficult for us to see the slow trend in this domain.

Generally there is a trade off between frequency domain analysis and time domain analysis. Hence various nature in both domain analyses is investigated. In this paper Sign-only syntheses of orthogonal transforms are presented. Sign-only synthesis is defined as the inverse transform of only the coefficient's signs and is derived from "Phase-only synthesis".^(1,2)

Phase-only synthesis is the inverse transform of only the phase of the signal Fourier transform. The Fourier

phase is actually continuous but the sign of the real orthogonal transform is just one bit, that's + or -.

In the real orthogonal transform the sign of the transform is just corresponding to the Fourier phase in the Fourier transform. Hence it looks very interesting to know that the Sign-only synthesis has similar nature to that of the phase-only synthesis.

Here we employ Discrete Cosine Transform (DCT), Hadamard Transform (HT), Karhunen-Loeve Transform (KLT), and Fourier Transform with binarized phase (BFT) as the real orthogonal transforms.

In section 2 the review of these typical orthogonal transforms is shown. The main theme "Sign-only synthesis" is revealed in section 3. The character of Sign-only synthesis is expressed here. Section 4 shows several simulation examples. Finally concluding remark with application of Sign-only synthesis is shown in section 5.

2 Real Orthogonal Transform

Here we present several definitions of orthogonal transforms employed in this paper.

2.1 Discrete Cosine transform

DCT is one of typical and most famous orthogonal transforms in digital image compression. This transform has a strong point that no marginal effect appears for an image compression.

2D DCT

$$F(u, v) = \frac{2C(u)C(v)}{\sqrt{N_x N_y}} \sum_{x=0}^{N_x-1} \sum_{y=0}^{N_y-1} f(x, y) \cos \frac{(2x+1)u\pi}{2N_x} \cos \frac{(2y+1)v\pi}{2N_y} \quad (1)$$

2D Inverse DCT

$$F(x, y) = \frac{2}{\sqrt{N_x N_y}} \sum_{u=0}^{N_x-1} \sum_{v=0}^{N_y-1} C(u)C(v)F(u, v) \cos \frac{(2x+1)u\pi}{2N_x} \cos \frac{(2y+1)v\pi}{2N_y} \quad (2)$$

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$$C(u), C(v) = \begin{cases} \frac{1}{\sqrt{2}} & u, v = 0 \\ 1 & u = 1, 2, \dots, N_x - 1 \\ & v = 1, 2, \dots, N_y - 1 \end{cases}$$

2.2 Hadamard Transform

Hadamard transform is transform using rectangular orthogonal functions. This has a very simple form.

Hadamard Transform

$$[Y_{MN}] = [H_M] [X_{MN}] [H_N] \quad (3)$$

Inverse Hadamard Transform

$$[X_{MN}] = [H_M] [Y_{MN}] [H_N] \quad (4)$$

$$[H_2] = \frac{1}{\sqrt{2}} \begin{bmatrix} H_n & H_n \\ H_n & -H_n \end{bmatrix} \quad (5)$$

$$[H_2] = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \quad (6)$$

Where $[H_2]$ is a transform matrix with 2×2 .

2.3 Karhunen-Loeve Transform (KLT)

Karhunen-Loeve Transform is well known as the best transform from the view point of minimum mean square error. But the transform kernel function is not predetermined but dependent upon the signal, hence it must be determined in each case.⁽²⁾

K-L Transform

$$F(u, v) = \sum_{m=0}^{N-1} \sum_{n=0}^{N-1} f(m, n) \varphi^{*(u,v)}(m, n) \quad (7)$$

Where $\varphi^{(u,v)}(m, n)$ is from the eigen vectors of the auto-covariance matrix of the signal $f(m, n)$. φ^* is a complex conjugate of φ .

Inverse Transform

$$F(m, n) = \sum_{u=0}^{N-1} \sum_{v=0}^{N-1} F(u, v) \varphi^{(u,v)}(m, n) \quad (8)$$

2.4 Fourier Transform

Fourier transform is the most typical and well known one but it is not real transform. We modify this a real one in section 3.5. Here, for simplicity we employ the continuous form:

Fourier transform

$$F(\omega_x, \omega_y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) e^{-j\omega_x x} e^{-j\omega_y y} dx dy \quad (9)$$

Inverse Fourier transform

$$f(x, y) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(\omega_x, \omega_y) \cdot e^{j\omega_x x} e^{j\omega_y y} d\omega_x d\omega_y \quad (10)$$

3 Sign-Only Stnthesis

Sign-only synthesis comes from phase-only synthesis in Fourier transform. Hence in this section we show a phase-only synthesis first.

3.1 Phase-only synthesis

Phase-only synthesis is defined as

$$\varphi(x, y) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{j\theta(\omega_x, \omega_y)} \cdot e^{j(\omega_x x + \omega_y y)} d\omega_x d\omega_y \quad (11)$$

Where $\theta(\omega_x, \omega_y)$ is the phase of the Fourier transform of the signal $f(x, y)$:

$$F(\omega_x, \omega_y) = |F(\omega_x, \omega_y)| e^{j\theta(\omega_x, \omega_y)} \quad (12)$$

Figure.1 is an original image with $256 \times 256 \times 8$ bit. Figure.2 is the phase-only synthesis of Fig.1. From these figures we see that phase-only synthesis has high pass nature.

Figure 1: Original image

Figure 2: Phase-only synthesis

3.2 DCT Sign-only synthesis

The DCT sign-only synthesis is defined as

$$g(x, y) = \frac{2}{\sqrt{N_x N_y}} \sum_{u=0}^{N_x-1} \sum_{v=0}^{N_y-1} C(u)C(v)e^{j\theta_{uv}} \cdot \cos \frac{(2x+1)u\pi}{2N_x} \cos \frac{(2y+1)v\pi}{2N_y} \quad (13)$$

where,

$$\theta_{uv} = \begin{cases} 0 & F(u, v) \geq 0 \\ \pi & F(u, v) < 0 \end{cases}$$

Comparing Fig.3 with Fig.2 the nature of DCT Sign-

Figure 3: DCT Sign-only synthesis

only synthesis is similar to that of phase-only synthesis.

3.3 Hadamard sign-only synthesis

The definition of two dimensional Hadamard transform is as follows: The Hadamard sign-only synthesis is defined as

$$[G_{MN}] = [H_M] [Z_{MN}] [H_N] \quad (14)$$

where Z_{MN} is vector with its element +1 or -1 and

$$Z_{MN} = \begin{cases} 1 & Y_{M,N} \geq 0 \\ -1 & Y_{M,N} < 0 \end{cases} \quad (15)$$

Hadamard Sign-only synthesis looks also similar to that of DCT one.

3.4 KLT sign-only synthesis

The definition of K-L transform and its sign-only synthesis are as follows:

$$g(m, n) = \sum_{u=0}^{N-1} \sum_{v=0}^{N-1} Z(u, v) \varphi^{(u, v)}(m, n) \quad (16)$$

where,

$$Z(u, v) = \begin{cases} 1 & F(u, v) \geq 0 \\ -1 & F(u, v) < 0 \end{cases}$$

Figure 4: Hardmard Sign-only synthesis

Figure 5: Karhunen-Loeve sign-only synthesis

KLT Sign-only synthesis has different nature with others. This is because the K-L transform function depends strongly upon the original signal. From Fig.5 KLT Sign-only synthesis is rather similar to the original image.

3.5 Binarized Fourier transform Sign-only synthesis

$$\varphi_{BFT}(x, y) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{j\theta_B(\omega_x, \omega_y)} \cdot e^{j(\omega_x x + \omega_y y)} d\omega_x d\omega_y \quad (17)$$

where,

$$\theta_B(\omega_x, \omega_y) = \begin{cases} \frac{\pi}{4} & -\frac{\pi}{4} \leq \theta(\omega_x, \omega_y) \leq \frac{3}{4}\pi \\ -\frac{3}{4}\pi & \frac{3}{4}\pi < \theta(\omega_x, \omega_y) \leq \pi \\ -\frac{\pi}{4} & -\pi < \theta(\omega_x, \omega_y) \leq -\frac{\pi}{4} \end{cases} \quad (18)$$

BFT Sign-only synthesis is just the same form of phase-only synthesis except $\theta(\omega_x, \omega_y)$. $\theta(\omega_x, \omega_y)$ is replaced by $\theta_B(\omega_x, \omega_y)$.

Figure 6: Binarized Fourier sign-only synthesis

4 Similarity To The First And Second Difference Of The Image

As shown above every Sign-only synthesis except KLT one looks similar to the first and/or the second difference of the original image. Hence we show the similarity with the first and the second difference of the image, and further the original image itself. The similarity is evaluated through the cross correlation: Evaluated value (EV) is defined as

$$EV = \text{MAX} \left[\frac{\sum \sum \varphi(i, j) \cdot D(i - i_1, j - j_1)}{\sigma_\varphi \sigma_D} \right] \quad (19)$$

$$(-2 \leq i_1, j_1 \leq 2)$$

$\varphi(i, j)$: Sign-only synthesis

$D(i - i_1, j - j_1)$: first or second difference of the original image, or original image itself

σ_φ : standard deviation of $\varphi(i, j)$

σ_D : standard deviation of $D(i, j)$

EV is defined as the max value of Eq.1 for (i_1, j_1) . These results are shown in Table 1. From Table 1 we see that every SOS is most similar to the second difference image except that of KLT. KLT-SOS is similar to the original image itself. This is because KLT-kernel function is made from the original signal, that is, signal dependent.

5 Simulation

In this section comparing the another examples of the Sign-only syntheses each other, we try to research further the nature of Sign-only synthesis. Figure.7 is the original image which includes a little bit higher frequency components. Figures 8~11 are the Sign-only synthesis of DCT, HT, KLT and Binarized FT respectively. Here also all figures are similar except the KLT SOS. This means that for all image DCT, HT and BFT have similar nature to the first and/or the second difference of the image. Only KLT has special nature. Even the Sign-only synthesis looks a little similar to the original image for Karhunen-Loeve Transform.

Table 1: Evaluation Value

SOS	first difference	second difference	original image
φ_{DCT}	0.325842	0.599882	0.262099
φ_{HT}	0.265198	0.487944	0.312060
φ_{KLT}	0.113076	0.227046	0.473443
φ_{BFT}	0.280549	0.491954	0.215192

Figure 7: Original image

6 Application To Face Recognition

The application fields of Sign-only synthesis is pretty wide because the required bit for one pixel in the Sign plane is only one (the least bit).⁽³⁾ This means we can transform every image into one bit/pixel image keeping the origin image edges.^(4,5) Here we present one application for face authentication.^(6,7) As described above the Sign-only synthesis gives the difference-like image. Hence in the transformed plane the plus and minus signs and its locations are very important for the image. Ac-

Figure 8: DCT Sign-only synthesis

Figure 9: Hardmard Sign-only synthesis

Figure 10: Karhunene-Loeve Sign-only synthesis

tually the plus and minus pattern is unique for the image. Then taking the significant point signs and rearranging the signs as $+1$ or -1 in one line we can make a simple binary sequence like $1, -1, -1, 1, 1, 1, -1 \dots$. If we make $+1$ correspond to white and -1 to black then a bar-code like signal will be constructed. We call it here "Bar-code" for simplicity.

6.1 Procedure for Face Authentication

Figure 12 shows a procedure for the face authentication. First we catch the face data into computer. The size of the face data is 256×256 pixels. It is enough for face authentication because the more pixels the more unstable in the authentication. The gray level is 256 i.e. 8bits. The procedure starts the rotation of the input face image. In this study the main axis of the face is required to be perpendicular to the horizontal line of the image. Rotating by 1 degree-step between ± 5 degrees through Hough transform and reflecting one side (for example, right side) to the other side (left side) at the central perpendicular line of the image, the matching level is calculated through cross correlation between both half sides of the face. After that one pixel shift to the right(or left) is executed between ± 5 pixels(to the right or left). And the same rotation procedure (i.e. axis detection procedure) is taken. The desired values of the shift and

Figure 11: Binarized Fourier Sign-only synthesis

Figure 12: Procedure of Face Authentication

rotation angle are decided at the point of the largest matching level(cross correlation value). Next stage is a size matching between the test face image(input image) and the stored image called "teacher image" here. This is also done by using partial cross correlation between these two images. The both eyes part of the teacher image(rectangular area) is utilized for the partial cross correlation. Figure 13 shows the rectangular area. The largest value of the cross correlation gives us the desired size and the matching position. The size changing is done for the test face image (only rectangular area). The coefficient of the changing size is between 0.8 and 1.2 with the 0.1 step. Consequently the test face image size can be changed into the same size as that of the teacher face. Next step is a gray scale normalization. Here the maximum gray level (white) is set as 255 and the minimum one is set as 0, i.e. 8 bit full dynamic range. Face clipping is the next stage. If the back ground of the face image is different from that of the teacher image, sometimes the back ground gives a serious defect in face authentication. Hence face clipping

Figure 13: Rectangular Area for cross correlation

shown in Fig.14 is needed. Inside in the clipping area (128×128 pixels) the authentication procedure should be done.

6.2 Application of DCT Sign-only Synthesis

As described in previous sections we can easily identify the image from the DCT Sign-only Synthesis. It means the DCT signs and the locations are unique for the image. Actually it is enough to take only characterized 100 signs (points) and the location in DCT of the image for identifying human face. Here we choose these 100 positions in DCT plane in reference to the same position amplitude i.e., amplitude order. When we take 100 positions the required bits are only $(1 \text{ Bit}(\text{sign}) + (7 \text{ Bit} + 7 \text{ Bit})(\text{Position})) \times 100 = 1,500$ bits. The number of fewer bits is the strong feature of this system.

Figure 14: Clipping Area

6.3 Simulation Examples

Figure 15 shows a simulation example. Figure 15(a) ~ 15(e) are different persons. And each bar code is corresponding with plus sign and the black bar corresponds to minus sign of the DCT. The order of the bar code is the same order of DCT amplitude in the teacher image. For example, at the position (3,1) in the DCT plane if the DCT amplitude is the biggest of all the first order of bar code is the DCT sign at the position (3,1). The second order of the bar code is the DCT sign at the position of the second biggest DCT example i.e., position

(0,0) is dropped from the first because DC element is always positive and the biggest one.

Authentication Procedure

- (1) Take a test face into computer through a Web camera or a video camera.
- (2) Much a size and position. This is done by using a rectangular area (see Fig.13) of the registrated teacher's face.
- (3) Bar code construction. The bar code is made from DCT signs as described above for the clipping area in Fig.14. The DCT signs should be taken at the same position on the DCT plane as the teacher's one.
- (4) Binary matching of the bar codes between these two codes of the trial face and the teacher's face.

Table 2: Authentication performance (Total 100 persons)

	Positive	Negative
True	473	27
False	1	

Table 2 shows the performance of this authentication system. "True" means a subject person and "False" means others. And "Positive" and "Negative" mean "accepted" and "unaccepted" respectively. Total number of faces is 500. 27 "True-Negatives" can be decreased at the coat of increasing "False ? Positive". There is a trade-off between "True-Negative" and "False-Positive" because human face has a lot of expressions. If the face image has an open mouth then several bars are changed from those of the straight face. Consequently from the straight face bar-code the same face with open mouth cannot be recognized. That means "True-Negative". In this case the threshold value for the recognition is set to be 93 for the bar code matching. If this threshold value is decreased then "True-Negative" will be decreased. "False-Positive", however, may be increased. Hence the threshold value should be determined optimally in each case. In fact several different bar codes for one person (different facial expression) are utilized to decrease "True-Negative". One "False Positive" in the Table 2 is due to twin brothers. They were resembled each other. The idea of this authentication system is an international patent of us and the products for entrance control and for computer log on control are made by one company.

Figure 15: Simulation Example

7 Conclusion

Sign-only synthesis of typical real orthogonal transforms has been presented. The nature of the sign-only synthesis is about the same for every transform except of K-L one. K-L sign-only synthesis is rather similar to the original image itself than the edge lines of the image. This is because the basis function in K-L transform is made from the original image itself, i.e., K-L transform is signal-dependent transform. Almost of all the sign-only syntheses have mainly the nature of a second difference of the signal. A second difference means an image edge detection nature. Consequently the signs of a real orthogonal transform play a very important role for the image construction. Generally the transform amplitude and the sign have the same important levels each other

from the information aspect. But actually it seems that the transform sign is more important than the transform amplitude for reconstructing the image because whenever the different image transform signs are utilized with keeping the transform amplitudes the resulting image is just close to the different image. That is to say, it is clear that the transform sign and its position characterize the image strongly in the real orthogonal transform. The merit of using a sign-only synthesis, in turn of the sign itself, is that the required bit is only one bit for the sign when the position of the sign is fixed. Hence we utilize the major signs on DCT plane for face authentication, where major sign means the amplitude of the same position as the position of the DCT sign is big. Now the product of this system is working for entrance control and computer log-on control. It looks that the application filed of real orthogonal transform sign is pretty wide. Our next stage is making police filing with barcode authentication.

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