Novel Delay-dependent Robust Stability Criteria of Delay Hopfield Neural Networks with Time-varying

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Abstract

This paper investigates the robust stability problem of Hopfield neural networks(HNNs) with time-varying delay. Two novel LMI-based delay-dependent robust stability criteria are obtained by constructing appropriate Lyapunov-Krasovskii functional. This new criteria based on the free-weighting matrices approach prove to be less conservativeness, which not only retain any useful terms in the derivative of Lyapunov-Krasovskii functional, but also consider the relationship among the time-delay, its upper bound and their difference. Finally, some numerical examples are given to demonstrate its merits and effectiveness of the proposed methods.

Keywords: Hopfield neural networks(HNNs), delay-dependent robust stability, time-varying delay, Lyapunov-Krasovskii functional, linear matrix inequality (LMI).

1. Introduction

During the past two decades, the Hopfield neural networks(HNNs) system has been attracted an increasing attention since it has found extensive applications in solving some optimization problems, system controlling, signal processing and static image treatment and other areas⁽¹⁻³⁾. It is well known that applications of neural networks heavily depend on their dynamic behavior, but time-delays often occur in the interaction between neurons related to their dynamic behaviors. Furthermore, the existence of time-delays is usually a source of instability and deteriorated performance. Therefore, the stability analysis of has not only important theoretical interesting, but also practical value.

Up to now, via different methods, lots of sufficient conditions have been obtained to guarantee the stability of delayed HNNs stability⁽⁴⁻¹⁹⁾. Among these works, stability criteria for HNNs can be classified into two types: delay-dependent criteria and delay-independent criteria. Since delay-dependent criteria make use of information on the length of delays, they are less conservative than delay-independent ones, especially when the time delay is small. Although delay-dependent stability results for the delayed HNNs has been obtained⁽¹⁶⁻¹⁹⁾, they have some conservativeness to extent, for example, the time delay is time-invariant. In practice, due to the finite switching speed of amplifiers or finite speed of information processing , time delay is often time-varying, which remains a space for further research.

Recently, a free-weighting matrix approach^(20,21) has been successfully employed to study the stability problem for neural networks and lots of less conservative conditions are obtained. Enlightened by it, in this paper, two improved LMI-based delay-dependent robust stability criteria are obtained for HNNs system with time-varying delay and constant delay by constructing appropriate Lyapunov-Krasovskii functional, which retain any useful terms in the derivative of Lyapunov-Krasovskii functional and consider the relationship among the time-delay, its upper bound and their difference. Finally, some numerical examples are given to demonstrate its advantages and effectiveness of the proposed methods.

Notation: Throughout this paper, the superscripts ‘T’ and ‘-1’ stand for the transpose and the inverse of a matrix; respectively; $\mathcal{R}^n$ denotes the n -dimensional Euclidean space; $P > 0$ means that the matrix P is positive definite; I is an appropriately dimensioned identity matrix; $\text{diag}\{\dots\}$ denotes a block-diagonal matrix; $\|z\|$ is the Euclidean norm of z ; and the symmetric matrix are denoted by *, e.g.,

$$\begin{bmatrix} X & Y \\ * & Z \end{bmatrix} = \begin{bmatrix} X & Y \\ Y^T & Z \end{bmatrix}.$$

2. Problem formulation

Consider the following delayed Hopfield neural networks(HNNs):

$$\dot{x}(t) = -Cx(t) + Ag(x(t-d(t))) + u \quad (1)$$

where $x(\cdot) = [x_1(\cdot), x_2(\cdot), \dots, x_n(\cdot)]^T \in \mathfrak{R}^n$ is the neuron state vector; $g(x(\cdot)) = [g(x_1(\cdot)), g(x_2(\cdot)), \dots, g(x_n(\cdot))]^T \in \mathfrak{R}^n$ denotes the neuron activation function; $u = [u_1, u_2, \dots, u_n]^T \in \mathfrak{R}^n$ is a constant input vector; $C = \text{diag}\{c_1, c_2, \dots, c_n\}$ is a diagonal matrix with $c_i > 0$; and A is the delayed connection weight matrix. $d(t)$ is the time delay. We will consider the following two cases for the time delay.

Case 1: $d(t)$ is a time-varying delay satisfying for all $t > 0$:

$$0 \leq d(t) \leq h \quad (2)$$

$$\dot{d}(t) \leq \mu \quad (3)$$

where μ and h are constants.

Case 2: $d(t)$ is a time-invariant delay satisfying for all $t > 0$:

$$0 \leq d(t) = d \leq h \quad (4)$$

where h is constant.

In addition, it is assumed that each neuron activation function in system (1), $g_j(\cdot)$, $j=1, 2, \dots, n$, satisfies the following condition:

$$0 \leq \frac{g_j(x) - g_j(y)}{x - y} \leq L_j, \quad \forall x, y \in \mathfrak{R}, x \neq y, j=1, 2, \dots, n \quad (5)$$

where L_j , $j=1, 2, \dots, n$ are positive constants.

Let a vector $x^* = [x_1^*, x_2^*, \dots, x_n^*]^T$ be an equilibrium point of equation (1). By the transformation $z(\cdot) = x(\cdot) - x^*$, one can shift the equilibrium point x^* of equation (1) to the origin, and consequently, system (1) can be rearranged as

$$\dot{z}(t) = -Cz(t) + Af(x(t-d(t))) \quad (6)$$

where

$$z(\cdot) = [z_1(\cdot), z_2(\cdot), \dots, z_n(\cdot)]^T$$

is the state vector of the transformed system, and

$$f(z(\cdot)) = [f(z_1(\cdot)), f(z_2(\cdot)), \dots, f(z_n(\cdot))]^T$$

with

$$f_j(z_j(\cdot)) = g_j(z_j(\cdot) + z_j^*) - g_j(z_j^*), j=1, 2, \dots, n.$$

Note that the functions $f_j(\cdot)$, $j=1, 2, \dots, n$ satisfy the following condition:

$$0 \leq \frac{f_j(z_j)}{z_j} \leq L_j, \quad f_j(0) = 0, \quad \forall z_j \in \mathfrak{R}, j=1, 2, \dots, n \quad (7)$$

which is equivalent to

$$f_j(z_j)[f_j(z_j) - L_j z_j] \leq 0, \quad f_j(0) = 0, j=1, 2, \dots, n \quad (8)$$

The following lemmas will be employed to derive the new criterion.

Lemma 1: For any vectors $a, b \in \mathfrak{R}^n$, the inequality

$$2a^T b \leq a^T X a + b^T X^{-1} b \quad (9)$$

Lemma 2: [9] Assuming that (7) holds, then

$$\int_v^u [f_i(s) - f_j(s)] ds \leq [u - v][f_i(u) - f_i(v)], \quad j=1, 2, \dots, n. \quad (10)$$

3. Delay-dependent asymptotic stability criteria

In the following, two new asymptotic stability criteria are derived by the free-weighting matrix approach.

Theorem 1: Under case 1, for given scalars $h \geq 0$ and μ , the origin of system (6) with (7) and a time-varying delay satisfying conditions (2) and (3) is globally asymptotically stable if there exist $P = P^T > 0$,

$$Q_i = Q_i^T > 0, i=1, 2, 3, \quad Z = Z^T > 0, \quad X = \begin{bmatrix} X_{11} & X_{21} \\ * & X_{22} \end{bmatrix} \geq 0,$$

$$\Lambda = \text{diag}(\lambda_1, \lambda_2, \dots, \lambda_n), \quad R = \text{diag}(r_1, r_2, \dots, r_n),$$

$S = \text{diag}(s_1, s_2, \dots, s_n)$, and any appropriately dimensioned matrices $T_i, i=1, 2$, $N = [N_1^T N_2^T]^T$, and $M = [M_1^T M_2^T]^T$ such that the following LMIs are feasible:

$$\Phi = \begin{bmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} & -M_1 & LR & -T_1 A \\ * & \Phi_{22} & 0 & 0 & \Lambda & -T_2 A \\ * & * & \Phi_{33} & -M_2 & 0 & LS \\ * & * & * & -Q_3 & 0 & 0 \\ * & * & * & * & Q_2 - 2R & 0 \\ * & * & * & * & * & -(1-\mu)Q_2 - 2S \end{bmatrix} < 0 \quad (11)$$

$$\Psi_1 = \begin{bmatrix} X & N \\ * & Z \end{bmatrix} \geq 0 \quad (12)$$

$$\Psi_2 = \begin{bmatrix} X & M \\ * & Z \end{bmatrix} \geq 0 \quad (13)$$

where

$$\begin{aligned}
\Phi_{11} &= Q_1 + Q_3 + N_1 + N_1^T + T_1 C + C^T T_1^T + h X_{11} \\
\Phi_{12} &= P + T_1 + C^T T_2^T \\
\Phi_{13} &= -N_1 + N_2^T + M_1 + h X_{12} \\
\Phi_{22} &= h Z + T_2 + T_2^T \\
\Phi_{33} &= -(1-\mu)Q_1 - N_2 - N_2^T + M_2 + M_2^T + h X_{22} \\
L &= \text{diag}\{l_1, l_2, \dots, l_n\}.
\end{aligned}$$

Proof: Construct the following Lyapunov-Krasovskii functional candidate:

$$\begin{aligned}
V(z(t)) &= V_1(z(t)) + V_2(z(t)) + V_3(z(t)) \\
V_1(z(t)) &= z^T(t) P z(t) + 2 \sum_{i=1}^n \lambda_i \int_0^{z_j} f_i(s) ds \\
V_2(z(t)) &= \int_{t-d(t)}^t [z^T(s) Q_1 z(s) + f^T(z(s)) Q_2 f(z(s))] ds \\
V_3(z(t)) &= \int_{t-h}^t z^T(s) Q_3 z(s) ds \\
V_4(z(t)) &= \int_{-h}^0 \int_{t+\theta}^t \dot{z}^T(s) Z \dot{z}(s) ds d\theta
\end{aligned} \quad (14)$$

where $P = P^T > 0$, $Q_i = Q_i^T > 0, i = 1, 2, 3$, $Z = Z^T > 0$,

and $\Lambda = \text{diag}(\lambda_1, \lambda_2, \dots, \lambda_n) \geq 0$ are to be determined.

Then calculating the derivatives of $V_i(z(t)), i = 1, 2, 3$ defined in (14) along the trajectories of system (6) and using (15) yields:

$$\dot{V}_1(z(t)) = 2z^T(t) P \dot{z}(t) + 2f^T(z(t)) \Lambda \dot{z}(t) \quad (15)$$

$$\begin{aligned}
V_2(z(t)) &\leq [z^T(t) Q_1 z(t) + f^T(z(t)) Q_2 f(z(t))] \\
&\quad - (1-\mu)[z^T(t-d(t)) Q_1 z(t-d(t)) \\
&\quad + f^T(z(t-d(t))) Q_2 f(z(t-d(t)))]
\end{aligned} \quad (16)$$

$$V_3(z(t)) = z^T(t) Q_3 z(t) - z^T(t-h) Q_3 z(t-h) \quad (17)$$

$$\begin{aligned}
V_4(z(t)) &= h \dot{z}^T(t) Z \dot{z}(t) - \int_{t-d(t)}^t \dot{z}^T(s) Z \dot{z}(s) ds \\
&\quad + \int_{t-h}^{t-d(t)} \dot{z}^T(s) Z \dot{z}(s) ds
\end{aligned} \quad (18)$$

For any appropriately dimensioned matrices $T_i, i = 1, 2$, the following holds:

$$0 = 2[z^T(t) T_1 + \dot{z}^T(t) T_2][\dot{z}(t) + C z(t) - A f(x(t-d(t)))] \quad (19)$$

On the other hand, using the Leibniz-Newton formula, for any appropriately dimensioned matrices N, M , the following equations are true:

$$0 = 2\xi_1^T(t) N [z(t) - z(t-d(t)) - \int_{t-d(t)}^t \dot{z}(s) ds] \quad (20)$$

$$0 = 2\xi_1^T(t) M [z(t-d(t)) - z(t-h) - \int_{t-h}^{t-d(t)} \dot{z}(s) ds] \quad (21)$$

where

$$\xi_1(t) = [z^T(t), z^T(t-d(t))]^T.$$

In addition, for any semi-positive definite matrix $X = X^T = \begin{bmatrix} X_{11} & X_{12} \\ * & X_{22} \end{bmatrix} \geq 0$, the following equation holds:

$$\begin{aligned}
0 &= \int_{t-h}^t \xi_1^T(t) X \xi_1(t) ds - \int_{t-h}^t \xi_1^T(t) X \xi_1(t) ds \\
&= h \xi_1^T(t) X \xi_1(t) - \int_{t-d(t)}^t \xi_1^T(t) X \xi_1(t) ds + \int_{t-h}^{t-d(t)} \xi_1^T(t) X \xi_1(t) ds
\end{aligned} \quad (22)$$

For any $R = \text{diag}(r_1, r_2, \dots, r_n) \geq 0$ and $S = \text{diag}(s_1, s_2, \dots, s_n) \geq 0$, It is clear from (8) that the following inequality are true:

$$\begin{aligned}
0 &\leq 2z^T(t) L R f(z(t)) - 2f^T(z(t)) R f(z(t)) \\
&\quad + 2z^T(t-d(t)) L S f(z(t-d(t))) \\
&\quad - 2f^T(z(t-d(t))) S f(z(t-d(t))).
\end{aligned} \quad (23)$$

Thus, adding the terms on the right side of (19)-(23) to $\dot{V}(z(t))$ yields:

$$\begin{aligned}
\dot{V}(z(t)) &\leq 2z^T(t) P \dot{z}(t) + 2f^T(z(t)) \Lambda \dot{z}(t) \\
&\quad + [z^T(t)(Q_1 + Q_3)z(t) + f^T(z(t)) Q_2 f(z(t))] \\
&\quad - (1-\mu)[z^T(t-d(t)) Q_1 z(t-d(t)) \\
&\quad + f^T(z(t-d(t))) Q_2 f(z(t-d(t)))] \\
&\quad - z^T(t-h) Q_3 z(t-h) + h \dot{z}^T(t) Z \dot{z}(t) \\
&\quad - \int_{t-d(t)}^t \dot{z}^T(s) Z \dot{z}(s) ds + \int_{t-h}^{t-d(t)} \dot{z}^T(s) Z \dot{z}(s) ds \\
&\quad + \xi_1^T(t) \Phi \xi(t) - \int_{t-d(t)}^t \eta^T(t, s) \Psi_1 \eta(t, s) ds \\
&\quad - \int_{t-h}^{t-d(t)} \eta^T(t, s) \Psi_2 \eta(t, s) ds \\
&\quad + 2[z^T(t) T_1 + \dot{z}^T(t) T_2][\dot{z}(t) + C z(t) - A f(x(t-d(t)))] \\
&\quad + 2\xi_1^T(t) N [z(t) - z(t-d(t)) - \int_{t-d(t)}^t \dot{z}(s) ds] \\
&\quad + 2\xi_1^T(t) M [z(t-d(t)) - z(t-h) - \int_{t-h}^{t-d(t)} \dot{z}(s) ds] \\
&\quad + h \xi_1^T(t) X \xi_1(t) - \int_{t-d(t)}^t \xi_1^T(t) X \xi_1(t) ds \\
&\quad + \int_{t-h}^{t-d(t)} \xi_1^T(t) X \xi_1(t) ds \\
&\quad + 2z^T(t) L R f(z(t)) - 2f^T(z(t)) R f(z(t)) \\
&\quad + 2z^T(t-d(t)) L S f(z(t-d(t))) \\
&\quad - 2f^T(z(t-d(t))) S f(z(t-d(t)))
\end{aligned} \quad (24)$$

where

$$\xi(t) = [z^T(t), \dot{z}^T(t), z^T(t-d(t)), z^T(t-h), f^T(z(t)), f^T(z(t-d(t)))]^T$$

$$\eta(t, s) = [\xi_1^T(t), \dot{z}^T(s)]^T$$

and Φ and $\Psi_i, i=1,2$ are defined in(11)-(13), if $\Phi < 0$ and $\Psi_i \geq 0, i=1,2$, then $\dot{V}(z(t)) \leq 0$ for any $\xi(t) \neq 0$. This completes the proof.

Remark 1: Theorem 1 provides a new delay-dependent global asymptotic stability condition for the delayed HNNs in terms of an LMI, which can deal with the case for HNNs system with time-varying delay.

Now, we provide a stability condition for the HNN system under case 2.

Theorem 2: Under case 2, for given scalars $h \geq 0$, the origin of system (6) with (7) and a time delay satisfying conditions (4) is globally asymptotically stable if there exist, $P = P^T > 0, Q_i = Q_i^T > 0, i=1,2,3, Z = Z^T > 0$,

$$X = \begin{bmatrix} X_{11} & X_{21} \\ * & X_{22} \end{bmatrix} \geq 0, \Lambda = \text{diag}(\lambda_1, \lambda_2, \dots, \lambda_n),$$

$$R = \text{diag}(r_1, r_2, \dots, r_n), S = \text{diag}(s_1, s_2, \dots, s_n),$$

and any appropriately dimensioned matrices $T_i, i=1,2, N = [N_1^T N_2^T]^T$, and $M = [M_1^T M_2^T]^T$ such that the following LMIs are feasible:

$$\Phi = \begin{bmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} & -M_1 & LR & -T_1 A \\ * & \Phi_{22} & 0 & 0 & \Lambda & -T_2 A \\ * & * & \Phi_{33} & -M_2 & 0 & LS \\ * & * & * & -Q_3 & 0 & 0 \\ * & * & * & * & Q_2 - 2R & 0 \\ * & * & * & * & * & Q_2 - 2S \end{bmatrix} < 0 \quad (25)$$

$$\Psi_1 = \begin{bmatrix} X & N \\ * & Z \end{bmatrix} \geq 0 \quad (26)$$

$$\Psi_2 = \begin{bmatrix} X & M \\ * & Z \end{bmatrix} \geq 0 \quad (27)$$

where

$$\Phi_{11} = Q_1 + Q_3 + N_1 + N_1^T + T_1 C + C^T T_1^T + h X_{11}$$

$$\Phi_{12} = P + T_1 + C^T T_2^T$$

$$\Phi_{13} = -N_1 + N_2^T + M_1 + h X_{12}$$

$$\Phi_{22} = h Z + T_2 + T_2^T$$

$$\Phi_{33} = Q_1 - N_2 - N_2^T + M_2 + M_2^T + h X_{22}$$

$$L = \text{diag}\{l_1, l_2, \dots, l_n\}.$$

Proof: Construct the following Lyapunov-Krasovskii functional candidate:

$$V(z(t)) = V_1(z(t)) + V_2(z(t)) + V_3(z(t)) + V_4(z(t))$$

$$V_1(z(t)) = z^T(t) P z(t) + 2 \sum_{i=1}^n \lambda_i \int_0^{z_i} f_i(s) ds$$

$$V_2(z(t)) = \int_{t-d}^t [z^T(s) Q_1 z(s) + f^T(z(s)) Q_2 f(z(s))] ds \quad (28)$$

$$V_3(z(t)) = \int_{t-h}^t z^T(s) Q_3 z(s) ds$$

$$V_4(z(t)) = \int_{-h}^0 \int_{t+\theta}^t \dot{z}^T(s) Z \dot{z}(s) ds d\theta$$

where $P = P^T > 0, Q_i = Q_i^T > 0, i=1,2,3, Z = Z^T > 0$, and $\Lambda = \text{diag}(\lambda_1, \lambda_2, \dots, \lambda_n) \geq 0$ are to be determined. Then following the similar line in Theorem 1, we can obtain Theorem 2.

Remark 2: Theorem 2 provides a new delay-dependent global asymptotic stability condition for the HNNs with constant delay in terms of an LMI. This is derived by constructing an appropriate Lyapunov-Krasovskii functional different from those papers^(18,19), and in the derivative process of Lyapunov-Krasovskii functional, it retains any useful terms and considers the relationship among the time-delay, its upper bound and their difference.

4. Numerical examples

In this section, two numerical examples are given to demonstrate the effectiveness of the proposed method.

Example 1: Consider the following delayed HNNs system(1)

$$\begin{aligned} \dot{x}_1(t) &= -0.8x_1(t) + 0.1g(x_1(t-d(t))) + 0.3g(x_2(t-d(t))) + 5 \\ \dot{x}_2(t) &= -0.8x_2(t) + 0.9g(x_1(t-d(t))) + 0.1g(x_2(t-d(t))) + 3 \end{aligned}$$

where the activation function is described by

$$g(x) = 0.5(|x+1| - |x-1|).$$

Then

$$C = \begin{bmatrix} 0.8 & 0 \\ 0 & 5.3 \end{bmatrix} \quad A = \begin{bmatrix} 0.1 & 0.3 \\ 0.9 & 0.1 \end{bmatrix} \quad L_1 = L_2 = 1.$$

When time delay is time-invariant, this example is discussed in both of papers^(16,17). Employing the LMIs^(16,17), the maxium delay h that guarantees the asymptotic stability of HNN (1) are obtained to be 0.058 and 2. However, by the Theorem 2 of this paper, it is found that for any arbitrarily large h (as long as the numerical computation is reliable), HNN system (1) is stable. So it can be seem this proposed method has a less conservativeness.

On the other hand, wen time delay is time-varying, both of papers^(16,17) cannot be used to determine whether HNN

system(1) is stable or not. But using the the Theorem 1 of this paper, we can also calculate the values of the maxium delay h which guarantees the asymptotic stability of HNN (1). The values of h for various μ are listed in the following Table 1.

TABLE 1

μ	0.8	0.9	2
Theorem 1	arbitrarily value	14.660	3.091

Example 2: Consider a delay HNN system in (6) with parameters:

$$C = \begin{bmatrix} 4.1989 & 0 & 0 \\ 0 & 0.7160 & 0 \\ 0 & 0 & 1.9985 \end{bmatrix}$$

$$A = \begin{bmatrix} -0.1052 & -0.5069 & -0.1121 \\ -0.0257 & -0.2808 & 0.0212 \\ 0.1205 & -0.2153 & 0.1315 \end{bmatrix}$$

$$L_1 = 0.4219 \quad L_2 = 3.8993 \quad L_3 = 1.0160$$

For this example, the delay-dependent condition⁽¹²⁾ cannot be satisfied for any arbitrarily small h . By the method^(18,19), the maxium delay h that guarantees the asymptotic stability of HNN system(6) are obtained to be 1.7484 and 1.7644, respectively, while we can obtain the delayed HNN system(6) is asymptotically stale under the condition $h \leq 2.1423$ by Theorem 2. So our result has less conservativeness.

5. Conclusions

In this paper, by employing the free-weighting matrices approach, two less conservative LMIs-based asymptotic stability criteria are obtained without ignoring any useful terms in the derivative of Lyapunov functional for neural networks with time delay. Finally, two numerical examples are given to demonstrate that the proposed method is an improvement over the existing ones.

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