Induced Charge Electrophoresis of a Conducting Cylinder in a Cylindrical Chamber

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Abstract

Induced charge electrophoresis (ICEP) of a conducting cylinder suspended in a cylindrical chamber is analytically studied in this paper. The results show that microvortices are generated around the cylinder, which is useful for fluid mixing and/or regulating in micro/nanofluidics. Furthermore, when the cylinder is off the center of the cylindrical chamber, it is driven into motion by the dielectrophoretic (DEP) force due to the non-uniform electric field and the ICEP force due to the asymmetric fluid flow. We discover that the cylinder not only translates but also rotates. By prohibiting the cylinder from translation but not rotation, a micro-motor could be generated based on the ICEP rotation of the cylinder. The velocities increase as the eccentricity increases and obtain peak values with the variation of the radius ratio.

Keywords: induced charge electrophoresis, microvortex generation, micro-motor.

1. Introduction

Electrokinetic phenomena occurred on the polarizable (conducting/dielectric) particle/surface are due to the interactions between the polarization charges and the applied electric field⁽¹⁾. When the surrounding electric field is non-uniform, the dipoles in the polarizable particles experience an electric force, which drives the particle into motion. This kind of particle motion is referred to as dielectrophoresis (DEP). The ions in the electrolyte solution are attracted to the polarized surface, forming an induced electric double layer (EDL). The interactions between the induced EDL and the applied electric field produce a quadratic fluid flow referred to as Induced charge electroosmosis (ICEO)⁽²⁾. When the ICEO flow is

asymmetric, the particle moves via induced charge electrophoresis (ICEP)⁽³⁾.

Induced charge electrokinetic (ICEK) phenomena, including ICEO and ICEP, have been widely studied due to the promising potential applications in micro/nanofluidics⁽³⁾, such as mixing⁽⁴⁻⁶⁾, demixing⁽⁷⁾, pumping^(8, 9), particle manipulation^(10, 11) and motion generation⁽¹⁰⁾. ICEP motion of the conducting particles has attracted lots of attentions. Particle-particle interactions⁽¹²⁻¹⁵⁾ and particle-wall interactions⁽¹⁵⁻¹⁸⁾ have been analyzed under DC/AC electric field for spherical/cylindrical particles. In this paper, the motion of a conducting cylinder suspended in a cylindrical chamber due to ICEP and DEP is studied under bipolar coordinate system. The ICEO flow around the cylinder and the velocities of the cylinder are presented.

2. Mathematical Model

2.1 Problem statement

We hereby consider an infinite conducting cylinder suspended in an insulating cylindrical chamber filled by electrolyte solution. The EDL adjacent to the cylinder's surface is assumed to be much thinner than the radius of the cylinder and the distance between the cylinder and cylindrical chamber. We adopt bipolar coordinate system (τ, σ) to conveniently describe the complex geometry (Fig. 1). The relationship between the Cartesian coordinate and bipolar coordinate systems is given by⁽¹⁹⁾,

$$x = a \frac{\sinh \tau}{\cosh \tau - \cos \sigma}, \quad y = a \frac{\sin \sigma}{\cosh \tau - \cos \sigma} \quad (1)$$

where $-\infty < \tau < \infty, 0 < \sigma \leq 2\pi$; a is a positive constant in bipolar coordinates. We use $\tau = \tau_i$ and $\tau = \tau_o$ to represent the surfaces of cylinder and cylindrical chamber, respectively (Fig. 1).

Fig. 1 Schematic diagram of the cylinder suspended in a cylindrical chamber.

2.2 Governing equations

Subjecting the cylindrical chamber to an electrical potential $E_0 R_o \sin(\sigma + \sigma_0)$, where $E_0 R_i < k_B T / (ze)$, the cylinder is polarized immediately after the establishment of the electric field. The ions are attracted to the cylinder surface and an EDL is formed within a very short time ($\sim 10^{-4}$ s)⁽²⁾. The bulk solution outside the EDL remains electrically neutral. Therefore, the electrical potential in the bulk solution is governed by the Laplace equation,

$$\nabla^2 \phi = 0 \quad (2)$$

No electric field lines enter the EDL, thus no-flux condition is applied on the cylinder,

$$\mathbf{e}_\tau \cdot \nabla \phi = 0 \text{ at } \tau = \tau_i \quad (3)$$

The induced zeta potential ζ on the cylinder's surface depends on the external electric field by the following equation⁽²⁰⁾,

$$\zeta = -\phi_e + \phi_c \quad (4)$$

where ϕ_e is the external electrical potential, ϕ_c is an integral constant defined as $\phi_c = \int_S \phi_e dA / A$, where A is the entire surface of the cylinder.

The DEP force and moment per unit length exerted on the cylinder are⁽²¹⁾,

$$\mathbf{F}_{DEP} = \int_A (\Pi_e \cdot \mathbf{e}_n) dA \quad (5)$$

$$\mathbf{M}_{DEP} = R \int_A \mathbf{e}_n \times (\Pi_e \cdot \mathbf{e}_n) dA \quad (6)$$

where $\mathbf{e}_n$ is the unit vector normal to the cylinder surface; Π_e is the Maxwell stress tensor,

$$\Pi_e = \epsilon_w \left(\mathbf{E} \mathbf{E} - \frac{1}{2} E^2 \mathbf{I} \right) \quad (7)$$

Reynolds number is very small in ICEO flow⁽²⁾ and the fluid is incompressible. Thus the Stokes equations are applicable. The general solution of the stream function ϕ in bipolar coordinates suitable for the present case was given by Jeffery⁽²²⁾,

$$\begin{aligned} h\phi = & A_0 \cosh \tau + B_0 \tau (\cosh \tau - \cos \sigma) \\ & + C_0 \sinh \tau + D_0 \tau \sinh \tau + h_1 \tau \sin \sigma \\ & h \sum_{n=1}^{\infty} \{a_n \cosh(n+1)\tau + b_n \cosh(n-1)\tau \\ & c_n \sinh(n+1)\tau + d_n \sinh(n-1)\tau\} \cos n\sigma \\ & + h \sum_{n=1}^{\infty} \{e_n \cosh(n+1)\tau + f_n \cosh(n-1)\tau \\ & + h \sum_{n=1}^{\infty} \{e_n \cosh(n+1)\tau + f_n \cosh(n-1)\tau \\ & g_n \sinh(n+1)\tau + h_n \sinh(n-1)\tau\} \sin n\sigma \end{aligned} \quad (8)$$

where $h = (\cosh \tau - \cos \sigma) / a$.

The established electric field exerts an electrical force on the ions within the EDL, which drives the ions as well as the bulk fluid into motion. The slip velocity on the cylinder surface is defined by the Helmholtz-Smoluchowski formula,

$$\mathbf{u}_s = \frac{\epsilon_w \zeta}{\mu} E_{\parallel} \mathbf{e}_\tau \text{ at } \tau = \tau_i \quad (9)$$

where the zeta potential ζ and the electric field on and tangent to the cylinder surface $E_{\parallel}$ are determined by the established electric field.

No-slip boundary condition is applied on the cylindrical chamber,

$$\mathbf{u} = 0 \text{ at } \tau = \tau_o \quad (10)$$

The ICEP force and moment due to the surrounding ICEO flow can be obtained from Eq. (5) and Eq. (6) by replacing Maxwell stress tensor [Eq. (7)] with the viscous stress tensor, $\Pi_H = -p\mathbf{I} + \mu[\nabla \mathbf{u} + (\nabla \mathbf{u})^T]$. The drag force and moment due to the cylinder motion with velocity $U_x \mathbf{e}_x + U_y \mathbf{e}_y + \Omega R_i \mathbf{e}_\sigma$ can also be obtained through Eq. (5) and Eq. (6). The cylinder is in free suspension, thus the net force and moment exerted on it should vanish,

$$\mathbf{F}_{DEP} + \mathbf{F}_{ICEP} + \mathbf{F}_d = 0, \mathbf{M}_{DEP} + \mathbf{M}_{ICEP} + \mathbf{M}_d = 0 \quad (11)$$

Solving these equations together, the electric and fluid flow fields and the cylinder velocities are determined,

$$\begin{aligned} U_x = & \frac{1}{8} U_i \sinh \tau_i [(\tau_i - \tau_o) - \tanh(\tau_i - \tau_o) \\ & + 2 \tanh(\tau_i - \tau_o) \cos 2\sigma_0] / [\sinh^2 \tau_o \end{aligned} \quad (12)$$

$$\begin{aligned}
& \times \cosh^2(\tau_i - \tau_o)] \\
U_y = & -\frac{1}{8} U_i \cosh \tau_i \tanh(\tau_i - \tau_o) \sin 2\sigma_0 / [\sinh^2 \tau_o \\
& \times \cosh^2(\tau_i - \tau_o)] \quad (13)
\end{aligned}$$

$$\begin{aligned}
\Omega = & -\frac{1}{8} \frac{U_i}{R_i} \tanh(\tau_i - \tau_o) \sin 2\sigma_0 / [\sinh^2 \tau_o \\
& \times \cosh^2(\tau_i - \tau_o)] \quad (14)
\end{aligned}$$

The cylinder velocities are Trigonometric functions of the electric field phase angle σ_0 . Thus the cylinder velocities (both of the direction and magnitude) can be manipulated by changing σ_0 .

3. Results and Discussion

3.1 ICEO flow field

To facilitate the following analysis, we hereby introduce two parameters, radius ratio R_r and eccentricity ε ,

$$R_r = \frac{R_i}{R_o}, \quad \varepsilon = \frac{\tilde{\varepsilon}}{R_o - R_i} \quad (15)$$

Fig. 2 depicts the flow patterns around the stationary cylinder with $\sigma_0 = 0, R_r = 0.1, \varepsilon = 0.5$. The flow contains four counter-rotating vortices. Due to the eccentricity, the ICEO flow is no longer symmetric to the y-axis. The two vortices on the left hand side are smaller than the two on the right hand side. The asymmetric ICEO flow exerts hydrodynamic force and/or moment on the cylinder. When it is free to go, it would translate and/or rotate. Once the cylinder reaches the center of the cylindrical chamber (the cylinder and cylindrical chamber become concentric), the ICEO flow regains perfect symmetry. The cylinder experiences no force/moment and becomes stationary. The detailed studies on the concentric case can be referenced from Zhao and Bau⁽⁶⁾. The velocity scale U_i used in Fig. 2 is $U_i = \varepsilon_w E_0^2 R_i / \mu$.

Fig. 2 ICEO flow (streamlines) around the stationary conducting cylinder. The increment between the neighboring streamlines is $\varphi/(U_i R_i) = 0.2$. Here $\sigma_0 = 0$.

3.2 Cylinder velocities

Fig. 3 presents the variations of cylinder velocities with the radius ratio R_r at different eccentricities ε with electric field phase angle $\sigma_0 = \pi/6$. Under this condition, U_x is positive; U_y and Ω are negative. The directions of these velocities as well as the magnitudes can be manipulated by changing σ_0 . Fig. 3 clearly presents that all the cylinder velocities diminish to zero at both $R_r = 0$ and $R_r = 1.0$, and the magnitudes of the velocities monotonically increase as ε increases. The cylindrical chamber becomes very large when R_r approaches 0, so as the distance between the cylinder and cylindrical chamber. Hence the influence of the cylindrical chamber on the cylinder motion becomes negligible. Once R_r reaches 0 ($\tau_o = 0$), the cylindrical chamber becomes a planar plate wall. For detailed studies on this situation, please reference to Zhao and Bau⁽⁶⁾. The annulus space becomes extremely small when R_r approaches 1, hence the viscous effect increases tremendously. Thus the cylinder motion decreases or even vanishes. However, one should notice that the EDLs on the cylinder and cylindrical chamber might become interacting with each other when R_r approaches 1, which invalidates the thin EDL approximation. Such extreme situation is beyond the scope of the present model.

Fig. 3 Variations of cylinder velocities with the radius ratio R_r at different eccentricities ε . Here $\sigma_0 = \pi/6$.

Table 1. Typical parameters of the ICEP system for 10^{-5} M KCl solution.

Dielectric constant	ε_w	$7 \times 10^{-7} \text{ g m V}^{-2} \text{ s}^{-2}$
Viscosity	μ	$1 \text{ g m}^{-1} \text{ s}^{-1}$
Ion diffusivity	D_i	$10^{-9} \text{ m}^2 \text{ s}^{-1}$
Debye length	λ_D	100 nm
Cylinder radius	R_i	10 μm
Charging time	$t_c = \lambda_D R_i / D_i$	10^{-3} s

3.3 ICEP micro-motor

As shown in Fig. 3 (c), the cylinder rotates when it does not share the same center with the cylindrical chamber. Utilizing the typical parameters in Table 1 and applying an electric field of strength $E_0 = 10^3 \text{ Vm}^{-1}$, the translational velocity scale of the cylinder is small, $U_i = 7 \times 10^{-6} \text{ m s}^{-1}$. The motion time scale is $t_0 = R_i / U_i = 1.4 \text{ s}$, which is large compared to the charging time $t_c = 10^{-3} \text{ s}$ (Table 1). Thus we can assume that the translation and rotation of the cylinder do not affect the EDL establishment on the cylinder.

Prohibiting the cylinder from translation but not rotation, a micro-motor could be generated due to the ICEP rotation. The rotational velocity scale is $\Omega_i = \varepsilon_w E_0^2 / \mu$, which is free of the cylinder radius. The rotational velocity of the cylinder Ω_i is a quadratic function of the electric field strength E_0 (Fig. 4). At $E_0 = 8.45 \times 10^3 \text{ Vm}^{-1}$, the rotational velocity scale Ω_i is 50 rs^{-1} . The actual rotational velocity can be determined by choosing appropriate eccentricity ε and radius ratio R_r according to Fig. 3 (c). The larger the electric field strength E_0 , the greater the rotational velocity scale Ω_i . However, the weak applied electric field approximation [$E_0 R_i < k_B T / (ze)$] should be satisfied.

Fig. 4 Variation of rotational velocity scale Ω_i with respect to the electric field strength E_0 .

4. Conclusions

The present analytical model characterizes the cylinder motion due to ICEP and DEP. The results reveal that microvortices are generated around the cylinder, which is useful for fluid mixing and/or regulating in micro/nanofluidics. Furthermore, we find the cylinder not only translates but also rotates. Micro-motor could be produced by letting the cylinder rotate but not translate. Larger eccentricity provides greater velocities. As radius ratio increases, the velocities obtain peak values.

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