A Single Deteriorating System's Replacement Model with Inspections

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Abstract

This study considers a generalized replacement model for a deteriorating system in which failures can only be detected by inspection. The system is assumed to have two types of failures and is replaced at the occurrence of the Nth type I failure (minor failure), or the first type II failure (catastrophic failure), or at working age T, whichever occurs first. The probability of a type I or type II failure depends on the number of type I failures since the previous replacement. Such a system can be repaired after a type I failure, but is deteriorating stochastically. That is, the operating intervals are decreasing stochastically, whereas the durations of the repairs are increasing stochastically. Based on these assumptions, we determine the expected net cost rate.

Keywords: Stochastic deterioration, Non-homogeneous

Poisson process, Inspection.

1. Introduction

Most replacement models assume that the repaired system or machine is returned to an “as good as new” state (or called perfect repair). This assumption means that, upon repair, an item has the same life distribution as a new item. If we only consider operating time with a negligible repair time, a renewal process is obtained. If we consider the non-zero repair times which are independent and identically distributed (i.i.d.) random variables, then an alternating renewal process with up and down periods is obtained. However, for a repairable deteriorating system, it may be more realistic to assume that a failed item will return to an “as bad as old” state after being repaired. That is, if the life

distribution of a new item was F , then the item after repair will have the survival function $\bar{F}_t$, where t is the item's age at failure and $\bar{F}_t(x) = \bar{F}(t+x)/\bar{F}(t)$. This is also known as minimal repair (see Barlow and Proschan [1, 2] for more detail).

However, for some practical deteriorating systems, the system can be different from that described above. Lam [3] considered a machine maintenance problem where after each repair, the machine's operating time becomes stochastically shorter. On the other hand, to model the effect of aging, repair times become stochastically larger. Therefore, we consider a repair and replacement model with such a feature for analyzing deteriorating systems. Due to the non-increasing operating times after repairs, the system will eventually die out, so the total system life is finite. One possible way of modeling this kind of deteriorating systems is to use the non-homogeneous Poisson process (NHPP). Lam [3] formulated the other way to use geometric process replacement model.

Maintenance and replacement models are usually based on the assumption that failures are detected and repaired simultaneously. However, for some systems, failures may be only identified by inspection. Nakagawa et al. [4] summarized the policies for periodic and random inspection. Chen et al. [5] applied periodic and random inspection policies to computer systems. Cheng and Li [6] studied a geometric process repair model with inspections. In this study, we develop a model with multiple failure modes with a replacement condition for the system's working age.

The outline of the article is as follows. In section 2, the model is described, then the expected net cost per unit time is found. Finally, Section 3 concludes.

2. General model

2.1 Model description

The system considered is “stochastically deteriorating”, in the sense that successive operating intervals after repairs are stochastically decreasing and successive repair durations are stochastically increasing. We assume that the system is subject to two types of failures, which can be detected through inspection. Type I failure (minor failure) is fixed by repair, whereas type II failure (catastrophic failure) is removed by replacement. We consider a replacement policy as follows: the system will be replaced at the N th type I failure (minor failure) or first type II failure (catastrophic failure) or at the working age T , depending on whichever occurs first. The probability of type II failure depends on the number of minor failures since the last replacement.

Let M be the number of failures until the first type II failure. Let $\bar{P}_k = P(M > k)$. That is, $\bar{P}_k$ is the probability that the first k failures are type I. We assume throughout the paper that the domain of $\bar{P}_k$ is

$\{0, 1, 2, \dots\}$ and $1 = \bar{P}_0 \geq \bar{P}_1 \geq \dots$. We use the notation

$\{\bar{P}_k\}$ for a sequence of probabilities. The sequence $\{\bar{P}_k\}$

is supposed to be known. Let $p_k = P(M = k)$; then, we know that

$$p_k = P(M = k) = \bar{P}_{k-1} - \bar{P}_k = \bar{P}_{k-1} (1 - (\bar{P}_k / \bar{P}_{k-1})).$$

Hence, if the k th failure occurs, it is categorized as either a type I with probability $q_k = \bar{P}_k / \bar{P}_{k-1}$ or a type II with probability $\theta_k = 1 - (\bar{P}_k / \bar{P}_{k-1})$.

Let X_i be the operating interval of the system after the $(i-1)$ th repair. The cumulative distribution function (c.d.f.) of X_i is $H_i(t)$. We assume that $\{X_i, i = 1, 2, \dots\}$

forms a stochastically decreasing sequence

with decreasing means $E[X_i] = \lambda_i$. Let Y_i be the repair

duration after the detection of the i th failure. Further, we

assume that $\{Y_i, i = 1, 2, \dots\}$ forms a stochastically

increasing sequence with increasing means $E[Y_i] = \mu_i$.

Let $\gamma^{i-1}h$ be the time period between two successive inspections in the i th repair cycle where h is a real number greater than zero. Let G_i be the idle period of the system from i th failure occurrence to the next inspection instant. We can calculate $E[G_i]$ using the total probability formula as

$$E[G_i] = \gamma^{i-1}h \sum_{g=0}^{\infty} \bar{H}_i(g\gamma^{i-1}h) - E[X_i] \quad (1)$$

where $\bar{H}_i(g\gamma^{i-1}h) = 1 - H_i(g\gamma^{i-1}h)$. Let $Q_{(k,1)}$ be the number of inspections during the operating period of the system from the $(k-1)$ th repair to the replacement at the N th type I failure. We calculate the $E[Q_{(k,1)}]$ as follows:

$$E[Q_{(k,1)}] = \sum_{g=0}^{\infty} \bar{H}_k(g\gamma^{k-1}h) \quad (2)$$

Let $Q_{(k,2)}$ be the number of inspections during the operating period of the system from the $(k-1)$ th repair to the replacement at the first type II failure. We calculate $E[Q_{(k,2)}]$ as follows:

$$E[Q_{(k,2)}] = \sum_{g=0}^{\infty} \bar{H}_k(g\gamma^{k-1}h) \quad (3)$$

Let $Q_{(k,3)}$ be the number of inspections during the operating period of the system from the $(k-1)$ th repair to the replacement at the working age T . We calculate

$E[Q_{(k,3)}]$ as follows:

$$E[Q_{(k,3)}] = \sum_{i=1}^{k-1} \sum_{g=0}^{\infty} \bar{H}_i(g\gamma^{i-1}h) + \sum_{g=1}^{\infty} [F_{k-1}(T - g\gamma^{k-1}h) - P(\sum_{i=1}^{k-1} X_i = T - g\gamma^{k-1}h)] \quad (4)$$

We assume the following cost structure is imposed: the unit of repair cost rate is C_1 , the unit of reward rate for the operating period is C_2 , the replacement cost is C_3 , the unit of penalty cost rate for the idle period is C_4 , and each inspection cost is C_5 .

2.2 The expected cost per unit time

Let W_i be the length of the i th successive replacement cycle (renewal cycle), and R_i be the operational cost over the renewal interval W_i for $i = 1, 2, 3, \dots$. Hence, $\{(W_i, R_i)\}$ constitutes a renewal reward process. If $D(t)$ denotes the expected cost of

operating the unit over time interval $[0, t]$, then we have

$$\lim_{t \rightarrow \infty} \frac{D(t)}{t} = \frac{E[R_1]}{E[W_1]} \quad (5)$$

(see, e.g., Ross [7] p.52). We denote the right-hand side of Eq. (5) by $B(N, T, h)$.

For our model, we have

$$E[W_1] = \sum_{k=1}^N \bar{P}_{k-1} \int_0^T (F_{k-1}(t) - F_k(t)) dt + \sum_{k=1}^{N-1} \mu_k \bar{P}_k F_k(T) + \sum_{k=1}^N E[G_k] \bar{P}_{k-1} F_k(T) \quad (6)$$

and

$$E[R_1] = C_3 + C_1 \sum_{k=1}^{N-1} \mu_k \bar{P}_k F_k(T) + C_4 \sum_{k=1}^N E[G_k] \bar{P}_{k-1} F_k(T) + C_5 \left[\sum_{k=1}^N \sum_{g=0}^{\infty} \bar{H}_k(g\gamma^{k-1}h) \bar{P}_{k-1} F_k(T) + \sum_{k=1}^N \left[\sum_{g=1}^{\infty} (F_{k-1}(T - g\gamma^{k-1}h) - P(\sum_{i=1}^{k-1} X_i = T - g\gamma^{k-1}h)) \right] \bar{P}_{k-1} (F_{k-1}(T) - F_k(T)) \right] - C_2 \sum_{k=1}^N \bar{P}_{k-1} \int_0^T (F_{k-1}(t) - F_k(t)) dt \quad (7)$$

Hence, the expected net cost per unit time is given by

$$B(N, T, h) = \frac{\left\{ C_3 + C_1 \sum_{k=1}^{N-1} \mu_k \bar{P}_k F_k(T) + C_4 \sum_{k=1}^N E[G_k] \bar{P}_{k-1} F_k(T) + \sum_{k=1}^N \sum_{g=0}^{\infty} \bar{H}_k(g\gamma^{k-1}h) \bar{P}_{k-1} F_k(T) + \sum_{k=1}^N \left[\sum_{g=1}^{\infty} (F_{k-1}(T - g\gamma^{k-1}h) - P(\sum_{i=1}^{k-1} X_i = T - g\gamma^{k-1}h)) \right] \bar{P}_{k-1} (F_{k-1}(T) - F_k(T)) \right\}}{\sum_{k=1}^N \bar{P}_{k-1} \int_0^T (F_{k-1}(t) - F_k(t)) dt + \sum_{k=1}^{N-1} \mu_k \bar{P}_k F_k(T) + \sum_{k=1}^N E[G_k] \bar{P}_{k-1} F_k(T)} \quad (8)$$

3. Conclusions

In this study, we proposed a generalized replacement model for deteriorating systems with failures which can be detected only through inspections. The system is subject to two types of failures and is replaced at the N th type I failure (minor failure) or first type II failure (catastrophic failure) or at the working age T , whichever occurs first. The expected net cost rate was derived.

References

- (1) R.E. Barlow, and F. Proschan : "Mathematical theory of reliability", New York: Wiley 1965.
- (2) R.E. Barlow, and F. Proschan : "Statistical theory of reliability and life testing probability models", New York: Holt, Rinehart & Winston 1975.
- (3) Y. Lam : "A note on the optimal replacement problem", Advanced Applied Probability, 20, pp.479-482, 1988.
- (4) T. Nakagawa, S. Mizutani, and M. Chen : "A summary

of periodic and random inspection policies”, Reliability Engineering and System Safety, 95, pp.906-911, 2010.

- (5) M. Chen, C. Qian, and T. Nakagawa : “Periodic and random inspection policies for computer systems”, Communications in Computer and Information Science, 257, pp.346-353, 2011.
- (6) G.Q. Cheng, and L. Li : “A geometric process repair model with inspections and its optimization”, International Journal of Systems Science, 43, 9, pp.1650-1655, 2012.
- (7) S.M. Ross : “Applied probability models with optimization applications”, San Francisco: Holden-Day 1970.