Unstructured Zero-Order Energy FEM : a potential NVH tool

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Abstract

An unstructured zero-order energy finite element method (uEFEM⁰) formulation is presented to simulate the high-frequency behavior of plate structures in contact with acoustic enclosures, which is can be seen as a typical simplification of cars. In this formulation, the vibration energy of all the bending, longitudinal and shear wave fields in the plates and sound press level in the acoustic enclosure can be obtained. This uEFEM⁰ formulation is validated by comparing the results with those from Statistical Energy Analysis (SEA). Good correlations are observed and the potential effort of the uEFEM⁰ as a NVH tool in car industry is identified.

Keywords: High-frequency structural-acoustic problem, NVH, uEFEM⁰, Power Transfer Coefficients.

1. Introduction

Noise, vibration, and harshness (NVH), is the study and modification of the noise and vibration characteristics of vehicles, particularly cars and trucks. In order to reduce the sound pressure level (SPL) in the driving room, a power transfer analysis should be carried out. A part of vibrational energy is transferred from the source due to the structural vibration, which is usually predominately low-frequency, the finite element method (FEM) and boundary element method (BEM) are rational choices. Another part of energy is transferred by the acoustic media, and it is recognized as noise. For the high-frequency problems, the Statistical Energy Analysis (SEA), developed by Lyon⁽¹⁾, is the most common simulation method. However, because of SEA is based on the division of subsystems, a single-averaged energy value with respect to time and space in each subsystem is provided at the end, local modeling details are ignored, it is the inherent weakness of SEA. In order to

overcome this disadvantage, Nefske and Sung⁽²⁾ applied Energy Flow Analysis (EFA) on high-frequency vibration in beam, and developed the Energy FEM (EFEM), in which the primary variable is the time- and space- averaged energy density. Thus, a variation of energy density can be determined easily, it is efficient to find out the peaks of structural vibration energy level or SPL, which would be fruitful results in a NVH analysis. Otherwise, Wang⁽³⁾ used a piece-wise constant approximation of the energy differential equations and developed a Zero-order EFEM (EFEM⁰) which integrates the best features of both EFEM and SEA⁽⁴⁾. However, the EFEM⁰ is based on structured mesh so far, rectangle grids in plates and hexahedron in acoustic domains are required, thus it has difficulty in dealing with irregular shape problems.

In this paper, an unstructured EFEM⁰ (uEFEM⁰) is presented, which is a new formulation to simulate the high-frequency structural-acoustic analysis. A numerical examples is given to verify the new formulation.

2. Overview of uEFEM⁰

2.1 The Governing Differential Equation

The governing equation in structures and acoustic domains can be expressed as^(5,6)

$$-\frac{c_g^2}{\eta\omega}\nabla^2 e + \eta\omega e = \pi_{in} \quad (1)$$

It could be seen that it is a differential equation analogous to heat transfer equation, in which e is the time- and space-averaged energy density, which characterize the vibration behavior of the wave domain, and the power flow is transferred along the gradient direction of it. c_g is the group speed, η is the hysteresis damping factor, and π_{in}

is the power input density. In this paper, all the three wave domains (bending, longitudinal and shear) in the plates and acoustic wave domain in the acoustic enclosure are considered.

2.2 The Treatments for the Junctions

When the traveling wave arrives at a junction⁽⁷⁾, such as a variation of geometry or multiple components connected together, the wave reflects and refracts, causing that the energy is discontinuous at the junction. In this paper, there are two types of junctions, which are plate-plate (P-P) joint and plate-acoustic (P-A) joint.

As deduced by Dong et al⁽⁷⁾ the relationship between the power flow through the discontinuities and the energy density on the boundary of the discontinuities has the same form in both P-P and P-A joints, as

$$\mathbf{q} = (\mathbf{I} - \boldsymbol{\tau})(\mathbf{I} + \boldsymbol{\tau})^{-1} \mathbf{c}_g \mathbf{e} \quad (2)$$

where $\mathbf{I}$ is an identity matrix, $\mathbf{c}_g$ is a diagonal matrix formed by the group speeds of coupling waves, and $\boldsymbol{\tau}$ is the power transfer coefficient matrix. The dimensions of the matrices are 2×2 for P-A joints, but $3n \times 3n$ for P-P joints, where n is the number of plates the joint contains.

The coefficients in the matrix $\boldsymbol{\tau}$ for P-P joint and P-A joint have been deduced by an analytical method by Langley et al and Bitsie⁽⁶⁾.

2.3 The Implementation of uEFEM⁰

The discretization procedure of governing equation is realized by EFEM⁰ with unstructured mesh, which means that triangular grids for three types of waves are used in 2-D plates, and tetrahedral grids are used for acoustic wave in 3-D acoustic domains. When the grids are produced, each grid is recognized as a control volume, and the variables are cell-centered, which means they are stored at the centroids of each control volume. Thus, at the junctions, none duplicate nodes are needed as the conventional EFEM⁽⁸⁾, and the existing FEM mesh can be more easily used to perform an uEFEM⁰ analysis.

Notice that the governing equation, Eq.(1) can be integrated in an elemental control volume V_i , as

$$\int_{V_i} -\frac{c_g^2}{\eta\omega} \nabla^2 e dV + \int_{V_i} \eta\omega e dV = \int_{V_i} \pi_{in} dV \quad (3)$$

A simple piece-wise constant approximation in V_i is

applied for the latter two terms, expressed as

$$\int_{V_i} \eta\omega e dV = \eta_i \omega e_i V_i, \quad \int_{V_i} \pi_{in} dV = \Pi_{in}^i = \pi_{in}^i V_i \quad (4)$$

and by applying the divergence theorem, the first term can be transferred into the power flux through the common boundary of adjacent cells, as

$$\begin{aligned} \int_{V_i} -\frac{c_g^2}{\eta\omega} \nabla^2 e dV &= \oint_{S_i} -\frac{c_g^2}{\eta\omega} \nabla e \cdot \mathbf{n} dS \\ &\approx \sum_{j=1}^m \left(-\frac{c_g^2}{\eta\omega} \nabla e \cdot \mathbf{S}_{i,j} \right) = \sum_{j=1}^m Q_{i,j} \end{aligned} \quad (5)$$

where $Q_{i,j}$ represents the power flux from V_i to its j^{th} adjacent volume V_j , and m is the number of common boundaries.

In this paper, different calculating procedures are carried out to calculate $Q_{i,j}$ in 2-D and 3-D domains.

In 2-D domain, the $Q_{i,j}$ is calculated from

$$Q_{i,j} = q_{i,j} L_j h \quad (6)$$

Where $q_{i,j}$ is the power flux density through the common boundary $\overline{AB}$, with the length of L_j , and h is the thickness of the plate, as shown in Fig. 1.

Fig. 1. Volumes Z_i and Z_j .

By resolve the vector $\mathbf{S}_{i,j}$ and the gradient of e into two directions, from Z_i to Z_j and from A to B , the power flux density can be expressed as

$$q_{i,j} = -\frac{c_g^2}{\eta\omega} \left(\frac{e_{Z_j} - e_{Z_i}}{d_i + d_j} + b \frac{e_B - e_A}{L_j} \right) \quad (7)$$

Where the factor b can be calculated by

$$b = -\cos \theta / \sin \theta \quad (8)$$

And θ is the included angle of $\overrightarrow{Z_i Z_j}$ and $\overrightarrow{AB}$.

While in 3-D domain, the power flux is divided into three parts (Fig. 2), as

Fig. 2. Two adjacent control volumes in 3-D domain

$$\begin{aligned} Q_{i,j} &= -\frac{c^2}{\eta\omega} (\nabla e \cdot \mathbf{S})_{B,C,D} \\ &= -\frac{c^2}{\eta\omega} \{ (\nabla e \cdot \mathbf{S})_{B,C,E} + (\nabla e \cdot \mathbf{S})_{C,D,E} + (\nabla e \cdot \mathbf{S})_{D,B,E} \} \end{aligned} \quad (9)$$

Similar to the 2-D domain, by applying the continuity of power flow, the form of each part can be solved. For example, the first part can be written in the form of

$$\begin{aligned} (\nabla e \cdot \mathbf{S})_{B,C,E} &= \nabla e_{B,C,E} \cdot \mathbf{S}_{B,C,E} \\ &= X_{Z_i} e_{Z_i} + X_{Z_j} e_{Z_j} + X_B e_B + X_C e_C \end{aligned} \quad (10)$$

And the scale coefficients are

$$\begin{aligned} X_{Z_i} &= -\frac{1}{3V_{Z_i BCE}} \left(\mathbf{S}_{B,C,E} + \frac{A_{Z_i}}{A_E} \mathbf{S}_{C,B,Z_i} \right) \cdot \mathbf{S}_{B,C,E} \\ X_{Z_j} &= -\frac{1}{3V_{Z_j BCE}} \frac{A_{Z_j}}{A_E} \mathbf{S}_{C,B,Z_i} \cdot \mathbf{S}_{B,C,E} \\ X_B &= -\frac{1}{3V_{Z_i BCE}} \left(\mathbf{S}_{C,Z_i,E} + \frac{A_B}{A_E} \mathbf{S}_{C,B,Z_i} \right) \cdot \mathbf{S}_{B,C,E} \\ X_C &= -\frac{1}{3V_{Z_i BCE}} \left(\mathbf{S}_{B,E,Z_i} + \frac{A_C}{A_E} \mathbf{S}_{C,B,Z_i} \right) \cdot \mathbf{S}_{B,C,E} \end{aligned} \quad (11)$$

And

$$\begin{aligned} A_E &= \frac{\mathbf{S}_{C,B,Z_i} \cdot \mathbf{S}_{B,C,E}}{V_{Z_i BCE}} - \frac{\mathbf{S}_{C,Z_j,B} \cdot \mathbf{S}_{B,C,E}}{V_{Z_j BCE}} \\ A_B &= \frac{\mathbf{S}_{C,E,Z_j} \cdot \mathbf{S}_{B,C,E}}{V_{Z_j BCE}} - \frac{\mathbf{S}_{C,Z_i,E} \cdot \mathbf{S}_{B,C,E}}{V_{Z_i BCE}} \\ A_C &= \frac{\mathbf{S}_{B,Z_j,E} \cdot \mathbf{S}_{B,C,E}}{V_{Z_j BCE}} - \frac{\mathbf{S}_{B,E,Z_i} \cdot \mathbf{S}_{B,C,E}}{V_{Z_i BCE}} \\ A_{Z_j} &= \frac{\mathbf{S}_{C,B,E} \cdot \mathbf{S}_{B,C,E}}{V_{Z_j BCE}}, \quad A_{Z_i} = -\frac{\mathbf{S}_{B,C,E} \cdot \mathbf{S}_{B,C,E}}{V_{Z_i BCE}} \end{aligned} \quad (12)$$

In Eqs.(9)-(12), V_{abcd} stands for the volumn of a tetrahedron whose vertexes are a, b, c , and d , and $\mathbf{S}_{a,b,c}$ represents the normal vector of Δ_{abc} , the direction is determined by right hand rule.

From Eqs. (6) and (7), the power flux through one of the common boundary in 2-D domain, while in 3-D domain, it is defined by Eq. (9), in which any term can be solved using Eq.(10).

3. The Global Stiffness Equation

In the prior section, the discretization method of the governing equation is presented, and note that the discretization procedure are done four times in bending, longitudinal, shear wave domains in plate and acoustic wave in acoustic domain, and an global stiffness equation can be obtained as

$$\begin{bmatrix} \mathbf{BB} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{LL} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{SS} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{AA} \end{bmatrix} \begin{Bmatrix} \mathbf{e}_B \\ \mathbf{e}_L \\ \mathbf{e}_S \\ \mathbf{e}_A \end{Bmatrix} = \begin{Bmatrix} \mathbf{\Pi}_B \\ \mathbf{\Pi}_L \\ \mathbf{\Pi}_S \\ \mathbf{\Pi}_A \end{Bmatrix}. \quad (13)$$

And because of the junctions talked in section 2.2, the global stiffness equation should be revised as

$$\begin{bmatrix} \mathbf{BB} & \mathbf{BL} & \mathbf{BS} & \mathbf{BA} \\ \mathbf{LB} & \mathbf{LL} & \mathbf{LS} & \mathbf{0} \\ \mathbf{SB} & \mathbf{SL} & \mathbf{SS} & \mathbf{0} \\ \mathbf{AB} & \mathbf{0} & \mathbf{0} & \mathbf{AA} \end{bmatrix} \begin{Bmatrix} \mathbf{e}_B \\ \mathbf{e}_L \\ \mathbf{e}_S \\ \mathbf{e}_A \end{Bmatrix} = \begin{Bmatrix} \mathbf{\Pi}_B \\ \mathbf{\Pi}_L \\ \mathbf{\Pi}_S \\ \mathbf{\Pi}_A \end{Bmatrix} \quad (14)$$

4. Validation

In this section, a numerical examples are developed to confirm the validity of the proposed uEFEM⁰. To verify the accuracy of the result obtained from uEFEM⁰, the corresponding models are established by AutoSEA⁽⁹⁾, a commercial software of SEA.

4.1 Simplified Passenger Vehicle Model

A passenger vehicle is simplified to seven structural plates with an internal air cavity. The plates is made of aluminum, with the uniform thickness of $h = 5 \text{ mm}$, and the internal cavity is filled with air. The dimensions and mesh of the vehicle are both shown in Fig. 3. The model comprises 468 structural plate elements and 1220 acoustic air elements, with 79 P-P joints and 468 P-A joints. The analysis is carried out in thirteen 1/3 octaves covering from 1.25kHz to 20kHz, and the input powers, 0.25W each in any octave, are applied at about the positions of four wheels, which are (0.6,0.3), (0.6,1.2), (2.5,0.3) and (2.5,1.2) in the coordinate system XOY set on the base plate to simulate the loading by irregular road to the wheels. And in the SEA model, a 1W power is applied at the base plate. All the damping factor is set to 0.06 for plates and 0.005 for acoustic media. The side plates are examples of plates with irregular shape, which would be difficult to deal with by EFEM⁰ developed by Wang⁽³⁾, however easy by this presented uEFEM⁰.

Fig. 3 uEFEM⁰ model of simplified vehicle

Fig. 4 and Fig. 5 show the comparison of results from uEFEM⁰ and AutoSEA, including the bending energy of plates and SPL of the cavity. The spatial variation of the energy density is presented in Fig. 6.

Fig. 4 Comparison of energy in plates

Fig. 5 Comparison of energy in air cavity

Fig. 6 The energy density cloud chart at $f = 2.15\text{kHz}$. (a) Bending wave, (b) Longitudinal wave, (c) Shear wave, (d) Acoustic wave

From the analysis of the simplified vehicle, the following conclusions can be summarized:

- The total bending energy of the incident plate shows a good agreement, whereas the results in other plates are slightly deviated. The result of SPL fits well for these two methods.
- As observed in Fig. 6, the bending energy level is highest at the loading points and decreases as the

distance of the loading points increases. The in-plane energy takes its peak in the side plates, which have the same direction with the loading forces. This phenomenon explains the importance of the in-plane energy in the energy transmission in such structures including 'L' angles. The SPL shows a significant value at the corners between the base and side plates. This result is due to a number of plate elements with relatively high bending energy that radiate energy into the air cavity at those corners.

- (c) The simplified passenger vehicle application demonstrate how the uEFEM⁰ formulation can be utilized as a simulation tool for structural-acoustic car applications, and it can be further developed to be a NVH tool to serve as a supplement of SEA.

5. Conclusions

An unstructured Zero-Order EFEM formulation has been developed for structural-acoustic problem, and both bending and in-plane waves are considered in plate structure. The derived formulation is applied to predict the energy distributions of a simplified vehicle model with an internal air cavity, and validated by comparing the results obtained from conventional simulation method.

Compared to SEA, the developed uEFEM⁰ formulation takes the advantage in determining the energy distribution. Thus, it is easier to grasp the power flow in each subsystem, which means that it is a better method to do the power transfer analysis in high frequency domain.

References

- (1) Lyon, R. H., Statistical Energy Analysis of Dynamical Systems: Theory and Applications. MIT press, 1975.
- (2) Nefske, D. J. and Sung, S. H., Power Flow Finite Element Analysis of Dynamic Systems: Basic Theory and Application to Beams. Journal of Vibration Acoustics Stress and Reliability in Design, Vol. 111, No. 1, pp. 94–100, 1989.
- (3) Wang, S., High Frequency Energy Flow Analysis Methods: Numerical Implementation, Applications, and Verification. PhD thesis, Purdue University, 2000.
- (4) Moens, I., On the Use and the Validity of the Energy Finite Element Method for High Frequency Vibrations. Faculteit Toegepaste Wetenschappen, 2001.
- (5) Bouthier, O. M. and Bernhard, R. J., Simple-models of the Energetics of Transversely Vibrating Plates. Journal of Sound and Vibration, Vol. 182, No. 1, pp. 149–166, 1995.
- (6) Bitsie, F., The Structural-Acoustic Energy Finite Element Method and Energy Boundary Element Method. PhD thesis, Purdue University, 1996.
- (7) Dong, J., Choi, K. K., Wang, A. M., Zhang, W. G. and Vlahopoulos, N., Parametric Design Sensitivity Analysis of High-Frequency Structural-Acoustic Problems using Energy Finite Element Method. International Journal for Numerical Methods in Engineering, Vol. 621, No. 1, pp. 83–121, 2005.
- (8) N. Vlahopoulos, L. O. Garza-Rios, C. Mollo, Numerical implementation, validation, and marine applications of an energy finite element formulation, Journal of Ship Research Vol. 43, No. 3, pp. 143–156, 1999.
- (9) S. M. Clifton, AutoSEA user guide, 1993.