Adaptive P-D control for DC servomotor

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Abstract

In general, it is said that PD controller is sufficient for position control of DC servomotor. However, we need a certain amount of trial and error to tune the parameters. In this research, we propose an adaptive P-D control system and illustrate the effectiveness using numerical example.

Keywords: PID control, adaptive control, DC servomotor.

1. Introduction

DC motors are widely used in mobile robots. In the motor position control system, a PID controller is often employed because of its simplicity, and there are many papers related to the tuning method of PID gains.

This paper presents a PID auto-tuning controller used an adaptive mechanism for the position control system of a DC servo motor. Learning method used in this study is based on partial model matching method, and it is simple. Each gain of PID controller is decided by this adaptive method, using the error between the command position and the actual motor position and it guarantees the stability and robustness of the system. From simulation results, it is verified that the auto-tuning controller has a good performance.

2. DC servomotor model

In this paper, we consider an armature controlled DC servomotor in Fig. 1. Let $i(t)$ be the armature current and $u(t)$ be the voltage applied to the armature, the electrical differential equation for a DC servomotor can be written as:

$$u(t) = Ri(t) + L \frac{di(t)}{dt} + v_e(t) \quad (1)$$

where R and L and represent respectively the resistance and inductance of the armature brush. The voltage $v_e(t)$ induced

between the output of DC servomotor is given by

$$v_e(t) = k_e \omega(t) \quad (2)$$

where k_e represents the voltage constant. The torque $T(t)$ developed by the DC servomotor is proportional to the current in the winding.

$$\tau(t) = k_\tau i(t) \quad (3)$$

where k_τ is the torque constant. The DC servomotor torque can also be written as:

$$\tau(t) = J \frac{d\omega(t)}{dt} + B\omega(t) \quad (4)$$

where J and B are respectively the inertia moment and the friction coefficient of the motor and load. From Eq. (1)-(4), the transfer function of the DC servomotor $G(s)$ from input voltage $u(t)$ to output angle $y(t)$ is obtained by:

$$G(s) = \frac{k_\tau}{s \{ (R + Ls)(Js + B) + k_e k_\tau \}} \quad (5)$$

3. P-D control system

As partial model matching method proposed by Kitamori we consider designing P-D control system in Fig. 2. Preparatory for calculation, the transfer function $G(s)$ is rewritten as denominator series expression $H(s)$

$$y_0(t) = \frac{1}{H(s)} u(t) \quad (6)$$

$$H(s) = h_1 s + h_2 s^2 + h_3 s^3 \quad (7)$$

and each coefficient is calculated as:

$$h_1 = \frac{BR + k_e k_\tau}{k_\tau} \quad (8)$$

Fig. 1. DC servomotor.

$$h_2 = \frac{JR + BL}{k_\tau} \quad (9)$$

$$h_3 = \frac{JL}{k_\tau} \quad (10)$$

Now reference models are similarly given by denominator series expression.

$$y_M(t) = \frac{1}{H_M(s)} r(t) \quad (11)$$

$$H_M(s) = 1 + \sigma s + \alpha_2(\sigma s)^2 + \alpha_3(\sigma s)^3 \quad (12)$$

From block diagram in Fig. 2, in the case of no disturbance, $y(t)$ can track $y_M(t)$ using following parameters.

$$\sigma = \frac{\alpha_2 h_3}{\alpha_3 h_2} \quad (13)$$

$$K_P = \frac{h_2}{\alpha_2 \sigma^2} \quad (14)$$

$$K_D = K_P \sigma - h_1 \quad (15)$$

4. Adaptive P-D Control System

Because control parameters in Eq. (13)-(15) contain the parameters of controlled object, when we cannot get exact value, the controlled performance may degrade. We cannot also the influence of sensor noise or disturbance. To overcome this difficulty, we design a robust adaptive P-D system. From block diagram in Fig. 2, we can get model matching condition as:

Fig. 2. P-D control system.

$$H_M(s) = 1 + \frac{1}{K_P} [H(s) + K_D s] \quad (16)$$

and tracking error is expressed by:

$$\varepsilon(t) = \frac{l}{H_M(s)} [u(t) - \theta^T \zeta(t)] + v(t) \quad (17)$$

where

$$l = \frac{1}{K_P} \quad (18)$$

$$\theta = [K_P \quad K_D]^T \quad (19)$$

$$\zeta(t) = \left[e(t) \quad -\frac{s}{1 + \delta s} y(t) \right]^T \quad (20)$$

$$v(t) = \frac{l K_D \delta s^2}{(1 + \delta s) H_M(s)} y(t) + \frac{l H(s)}{H_M(s)} d(t) \quad (21)$$

When the plant parameters are unknown, true parameters are replaced by the adjustable parameters.

$$u(t) = \hat{\theta}^T(t) \zeta(t) \quad (22)$$

So we get an error equation as:

$$\varepsilon(t) = \frac{l}{H_M(s)} \tilde{\theta}^T(t) \zeta(t) + v(t) \quad (23)$$

However, this error transfer function $1/H_M(s)$ is no longer strictly positive real. A famous technique called error augmentation can be used to avoid this difficulty in finding an adaptation law for this error model. The basic idea of the technique is to consider a so-called augmented error which correlates to the parameter error in a more desirable way than the tracking error. The augmented error is computed from an auxiliary error. First, let us define an auxiliary error

$$\eta(t) = \tilde{\theta}^T(t) \left(\frac{1}{H_M(s)} \zeta(t) \right) - \frac{1}{H_M(s)} \left(\tilde{\theta}^T(t) \zeta(t) \right) \quad (24)$$

Now let us define an augmented error, by combining the tracking error with the auxiliary error as

$$\varepsilon'(t) = \varepsilon(t) + \hat{l}(t)\eta(t) \quad (25)$$

And we obtain

$$\varepsilon'(t) = l\tilde{\theta}^T(t)\xi(t) + \tilde{l}(t)\eta(t) + v(t) \quad (26)$$

$$\xi(t) = \frac{1}{H_M(s)} \zeta(t) \quad (27)$$

Using the normalizing signal and dead zone, the control parameters are updated by

$$\dot{\hat{\theta}}(t) = -\Gamma \xi_N(t) D(\varepsilon'_N(t)) \quad (28)$$

$$\dot{\hat{l}}(t) = -\gamma \eta_N^T(t) D(\varepsilon'_N(t)) \quad (29)$$

where

$$\varepsilon'_N(t) = \varepsilon'(t) / N(t) \quad (30)$$

$$\eta_N(t) = \eta(t) / N(t) \quad (31)$$

$$\xi_N(t) = \xi(t) / N(t) \quad (32)$$

$$N(t) = \rho + m(t) \quad (33)$$

$$m(t) = \frac{1}{s + \kappa} (|u(t)| + |y(t)|) \quad (34)$$

$$D(\varepsilon'_N(t)) = \begin{cases} \varepsilon'_N(t) - d_0 & \varepsilon'_N(t) > d_0 \\ 0 & |\varepsilon'_N(t)| \leq d_0 \\ \varepsilon'_N(t) + d_0 & \varepsilon'_N(t) < -d_0 \end{cases} \quad (35)$$

This adaptive system guarantees asymptotical trajectory tracking of output $y(t)$ in certain degree of disturbance or model uncertainty.

5. Numerical example

To show the effectiveness of proposed control system, we conducted a numerical simulation. We define controlled object:

$$G(s) = \frac{k_\tau}{s \{ (R + Ls)(Js + B) + k_e k_\tau \}}$$

Reference model:

$$G_M(s) = \frac{1}{1 + \sigma s + \alpha_2(\sigma s)^2 + \alpha_3(\sigma s)^3}$$

and employ the Adaptive P-D scheme where $R=1$, $L=0.01$, $J=10$, $B=0.001$, $k_e = k_\tau = 10$. Reference input $r(t)$ is a unit step input ($t > 1$) and disturbance $d(t)$ is added ($t > 5$), where design parameters are set as $\alpha_2=3/8$, $\alpha_3=1/16$, $\Gamma=10000I$, $\gamma=10000$, $\delta=0.01$, $\rho=0.001$, $\kappa=0.01$, $d_0=0.0001$. The control result is in Fig. 3 and we can confirm the convergence of output $y(t)$ and estimate value of $\theta(t)$.

6. Conclusions

In this paper, we propose an adaptive P-D control scheme for DC servomotor. The effectiveness of proposed method is confirmed by numerical simulation. We intend to present the proof of stability and experimental result in another conference.

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Fig. 3. Control Result.