Discrete PID $\times$ (n-2) Stage PD Cascade Controllers

Proposed by Kitti

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Abstract

A Proportional-Integral-Derivative (PID) Controller has only two zeros and not enough for most industrial plants that are third or higher order plant. To achieve the desired specifications by PID controls, Associate Professor Dr. Kitti Tirasesth proposed the way to add more zeros to the PID controller according to the order of the plant. By this technique, the designer find only two unknown parameters based on the root locus magnitude and angle conditions. By increasing only the controller gain, the faster response with smaller overshoot can be obtained. Now the discrete-time proposed controllers are ready for real time implementation.

Keywords: PID Controller, PD Controller, Root Locus.

1. Introduction

According to the type and plant's order, most industrial plants are type 0 and consist of three to five first order lags or dead time plus one first order lag⁽¹⁾. Then, the n^{th} order plant to be controlled here, its transfer function is assumed to be given as

$$G(s) = \frac{K_n}{s^N (T_1 s + 1)(T_2 s + 1) \cdots (T_p s + 1)}, \quad (1)$$

where, the order of the plant is $n = N + p$. However, the Proportional-Integral-Derivative (PID) controller is properly applied to the typical second order plant only. The difficulty of satisfying the desired specifications with PID controller for a third order plant can easily show by the following example.

Fig. 1. Control system structure.

Let the transfer function of the PID controller, the third order plant, and one of the dominant closed-loop pole are

$$K(s) = \frac{K_c (s + z_c)^2}{s},$$

$$G(s) = \frac{1}{(s + 1)(s + 3)(s + 6)}, \quad (2)$$

$$s_d = -6 + j6.$$

$$\angle K(s)G(s) = \pm(2k + 1)\pi, \quad k = 0, 1, 2, \dots$$

$$|K(s)G(s)| = 1 \quad (3)$$

Fig. 2. Find the angle and location for double z_c .

Fig. 3. Root Locus Plot for $K(s) = \frac{K_c(s + z_c)^2}{s}$.

Find the zeros of $(s + z_c)^2$ and the gain K_c of the PID controller $K(s)$ in Eqn. 2 from the Angle and Magnitude conditions of the Root Loci in Eqn. 3, yield

$$K(s) = \frac{K_c(s + z_c)^2}{s}, \quad (4)$$

$$= \frac{23.547(s - 2.791)^2}{s}.$$

From the root locus plot in Fig. 3, there are three interesting points. First, at $K_c = 23.547$, although the root loci pass through complex conjugate dominant closed-loop poles, but there are two closed-loop poles in the RHP (Right Half-Plane), the closed-loop system is unstable; the corresponding response is shown by red solid line. Second, at $K_c = 2.1666$, the root locus cross $j\omega$ axis, a sustained oscillation occurs as shown in green solid line. Third, at $K_c = 0.346$, this point is called “breakaway point”, the roots are both negative real and repeated in the LHP (Left Half-Plane), results in a critically damped response as shown in blue solid line in Fig. 4, respectively.

In this example, the plant is type 0, 3rd order. Note that if one $(3-2=1)$ more zero is available, the total angle for the zeros can be shared to each by smaller degrees such that they are able to arbitrarily place in LHP with the desired specifications. By this reason, the “PID $\times(n-2)$ stage PD cascade controller for SISO systems”⁽²⁾ had been proposed by Kitti ; Associate Professor Dr. at KMITL.

Fig. 4. Corresponding output responses.

Fig. 5. Digital Control system design steps.

2. Methodology

There are 2 steps as shown in Fig. 5, for the design procedure of control system as follows:

- 1) Plant modeling,
- 2) Controller design.

2.1 Continuous-Time System

The open loop transfer function $K(s)G(s)$ in Fig. 1, between the PID by $(n-2)$ PD Controller and the n th order plant can be written as follows:

$$K(s)G(s) = K \frac{\overbrace{(s + z_1)(s + z_2) \cdots (s + z_{pd})}^{(n-2) \text{ PD}}}{\underbrace{s \cdot s^N (s + p_1)(s + p_2) \cdots (s + p_p)}_{n^{\text{th}} \text{ order plant}}}. \quad (5)$$

The PID by $(n-2)$ PD Controller designed by using Root Locus Technique for the 3rd order plant has the transfer function as written in Eqn. 6, and the corresponding Root Locus Plot is shown in Fig. 6.

Fig. 6. Root Locus Plot for $K(s) = \frac{K_c(s + z_c)^3}{s}$.

Fig. 7. CT - Root Locus Plot for the Proposed by Kitt.

Fig. 8. CT - Unit step response for the Proposed by Kitt.

$$K(s) = \frac{K(s + z_1)(s + z_2)(s + z_{pd})}{s} = \frac{K_c(s + z_c)^3}{s}, \quad (6)$$

$$= \frac{12.065(s + 5.25)^3}{s}.$$

Fig. 9. Digital control system design techniques.

The solutions for the case of using the method as proposed by Kitt, the PID by $(n-2)$ PD controller's transfer function is given in Eqn. 7, the corresponding Root Locus Plot in Fig. 7, and a unit step response, in Fig. 8.

$$K(s) = \frac{K(s + z_1)(s + z_2)(s + z_{pd})}{s}, \quad (7)$$

$$= \frac{11.097(s + 3.1)(s + 6.1)(s + 6.364)}{s}.$$

Note that, the Root Locus Technique find z_1, z_2, z_{pd} and K , but the Proposed by Kitt find only z_{pd} and K .

2.2 Discrete-Time System

Recently there were two discretization method have been proposed for designing the Discrete-Time PID by $(n-2)$ PD Controllers. The first is use 'zoh'⁽³⁾, Zero Order Hold discretization method as shown in Fig. 9. While, the second use 'tustin'⁽⁴⁾, the "Tustin's Method", discretize the plant first to obtain a sampled-data system or discrete-time system.

The Discrete-Time PID by $(n-2)$ PD Controllers transfer functions by the 'zoh' and 'tustin' methods can be written by Eqn. 8 and Eqn. 9, respectively.

$$\left\{ \begin{array}{l} K_{PID}(z) = K_p + K_I \left(\frac{Tz}{z-1} \right) + K_D \left(\frac{z-1}{Tz} \right), \quad \text{'zoh'} \\ \equiv \frac{K_{pid}(z - z_1)(z - z_2)}{z(z-1)} \equiv \frac{K_{pid}(z - z_{pid})^2}{z(z-1)}, \\ K_{PD}(z) = \frac{(K_p + K_D)z - K_D}{z} \equiv \frac{K_{pd}(z - z_{pd})}{z}, \\ \overline{D(z)} \\ K_{(n-2)}(z) = \frac{K_{pid}(z - z_1)(z - z_2)}{z(z-1)} \times \frac{K_{pd}(z - z_{pd})}{z}. \end{array} \right. \quad (8)$$

$$\left\{ \begin{aligned}
K_{PID}(z) &= K_P + K_I \left\{ \frac{T}{2} \left(\frac{z+1}{z-1} \right) \right\} + K_D \left\{ \frac{2}{T} \left(\frac{z-1}{z+1} \right) \right\}, \\
&\equiv \frac{\alpha_2 z^2 + \alpha_1 z + \alpha_0}{(z+1)(z-1)}, \quad \text{'tustin'} \\
&\equiv \frac{K_{pid}(z-z_1)(z-z_2)}{(z+1)(z-1)} \equiv \frac{K_{pid}(z-z_{pid})^2}{(z+1)(z-1)}, \quad (9) \\
K_{PD}(z) &= K_P + K_D \left\{ \frac{2}{T} \left(\frac{z-1}{z+1} \right) \right\} \equiv \frac{K_{pd}(z-z_{pd})}{(z+1)}, \\
\frac{D(z)}{K_{(n-2)}(z)} &= \frac{K_{pid}(z-z_1)(z-z_2)}{(z+1)(z-1)} \times \frac{K_{pd}(z-z_{pd})}{(z+1)}.
\end{aligned} \right.$$

In case of using Tustin's Method, the corresponding discretized plant $G(z)$ with the sampling time $T=1/50$ sec/samples, the Discrete-Time PID by $(n-2)$ PD Controllers, $D(z)=K_{(n-2)}(z)=K(z)$ and the dominant closed-loop pole z_d are

$$\begin{aligned}
G(z) &= \frac{[10^{-5}](z+0.9991)(z+1.0005 \pm j0.0008)}{(z-0.9802)(z-0.9417)(z-0.8868)}, \\
&\quad \equiv (s+3) \quad \equiv (s+6) \\
&\stackrel{=?}{=} \frac{\text{Pre-Placed } \tilde{K}(z-z_1)(z-z_2)(z-z_{pd})}{(z+1)(z+1)(z-1)}, \quad (10) \\
z_d &= e^{T s_d} = 0.8805 + j0.1062.
\end{aligned}$$

Last time, the Proposed by Kitti has been applied to find $K(z)$ in Eqn. 10 in z -plane directly with the difficulty so much when the sampling time T is very small.

2.3 The Proposed by Kitti

This time, the Continuous-Time controller $K(s)$ in s -domain will be discretized to Discrete-Time controller $K(z)$ in z -domain as shown in Fig. 10.

Fig. 10. Discretization.

Rewrite Eqn. 7 in polynomial form, yield

$$\begin{aligned}
K(s) &= \frac{K(s+z_1)(s+z_2)(s+z_{pd})}{s}, \\
K(s) &= \frac{c_3 s^3 + c_2 s^2 + c_1 s + c_0}{s}, \quad \Leftarrow \quad (11)
\end{aligned}$$

$$\begin{aligned}
c_1 &= K(z_1 z_2 + z_{pd} z_1 + z_{pd} z_2), \quad c_3 = K, \\
c_2 &= K(z_1 + z_2 + z_{pd}), \quad c_0 = K z_1 z_2 z_{pd}.
\end{aligned}$$

Transforms the coefficients to obtain $K(z)$,

$$K(z) = \frac{d_3 z^3 + d_2 z^2 + d_1 z + d_0}{(z+1)(z+1)(z-1)}, \quad \Leftarrow \quad (12)$$

where

$$\begin{bmatrix} d_3 \\ d_2 \\ d_1 \\ d_0 \end{bmatrix} = \frac{1}{2T^2} \begin{bmatrix} 2T^2 & T^3 & 4T & 8 \\ 2T^2 & 3T^3 & -4T & -24 \\ -2T^2 & 3T^3 & -4T & 24 \\ -2T^2 & T^3 & 4T & -8 \end{bmatrix} \begin{bmatrix} c_1 \\ c_0 \\ c_2 \\ c_3 \end{bmatrix}. \quad (13)$$

Finally, the solutions to $K(z)$ in Eqn. 10 be

$$K(z) = \frac{\overbrace{[10^5]}^K \underbrace{(z-0.9399)}_{(s+3.1)} \underbrace{(z-0.8850)}_{(s+6.1)} \underbrace{(z-0.8803)}_{z_{pd}}}{(z+1)(z+1)(z-1)}. \quad (14)$$

From the solutions for $K(z)$ in Eqn. 14, it is evidently seen that, the controller's zeros z_1, z_2 are automatically placed in z -plane, and the root loci also pass through the dominant closed loop pole z_d with circular shape as same as in s -plane. The corresponding root locus in z -plane is shown in Fig. 11.

Fig. 11. DT - Root Locus Plot for the Proposed by Kitti.

Fig. 12. OCF block diagram realization.

Rewrite Eqn. 12 to obtain the Observer Canonical Form (OCF) block diagram.

$$K(z) = \frac{M(z)}{E(z)} = \frac{Y(z)}{U(z)} = \frac{b_3 z^3 + b_2 z^2 + b_1 z + b_0}{z^3 + a_2 z^2 + a_1 z + a_0}. \quad (15)$$

The state space model for the discrete-time controller block diagram in Fig. 12 is given by

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 0 & -a_0 \\ 1 & 0 & -a_1 \\ 0 & 1 & -a_2 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix} + \begin{bmatrix} b_0 - a_0 b_3 \\ b_1 - a_1 b_3 \\ b_2 - a_2 b_3 \end{bmatrix} e(k), \quad (16)$$

$$\underbrace{m(k)}_{y(k)} = \underbrace{x_3(k) + b_3 e(k)}_{u(k)}.$$

Eqn. 16 and block diagram in Fig. 12 may be used as a guideline for implementation of difference equations in real time.

The unit step responses shown in Fig. 13 are for the designed controller gain K and for $10K$ with faster response and smaller in percentage of the overshoot. So, the designer should use the controller's coefficients that corresponding to this value of the controller gain K .

Fig. 13. DT - Unit step responses for the Proposed by Kitti.

3. Conclusions

Since, the PID controller is suitable for a second order plant. But, for a third or higher order plant, the zeros provided by the PID controller is not enough. In order to give more zeros, the $PID \times (n-2)$ stage PD cascade controller is then proposed by Associate Professor Dr. Kitti Tirasesth. Starting from the continuous-time controller, follow by two more of the discrete-time controllers. This paper is the third version of the discrete-time controller that directly transformed from the continuous-time controller with all features is maintained and ready for real time implementation.

Acknowledgment

The authors thank Associate Professor Dr. Kitti Tirasesth very much for his kindly valued suggestions.

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