An Observer-Based Controller Design for a Markovian Jumping Singular System with Polytopic Uncertainties

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Abstract

In this paper, an observer-based controller design problem for a discrete-time Markovian jumping singular system (DMJSS) with polytopic uncertainties is considered, which is based on a set of necessary and sufficient conditions in terms of linear matrix inequalities (LMIs) for an admissibility analysis problem. A proposed mode dependent observer and a controller can be designed separately via two sets of derived LMI-based synthesis conditions. There are two principal contributions in this paper. One is that a set of strictly LMI-based necessary and sufficient conditions for the admissibility analysis problem for a DMJSS is extended to the case with uncertainties. Another one is that a suitable observer and a controller can be designed independently via two sets of strictly LMI-based synthesis conditions instead of a set of complex algorithm in terms of bilinear or nonconvex inequalities.

Keywords: Observer-Based Control, Markovian Jumping, Singular Systems.

1. Introduction

A singular system description can simultaneously take into account a system's dynamics and its algebraic constraints, which is more general than a normal state-space one. The theoretical developments for a singular system are worthwhile and there are more challenges, because the essential consideration for a singular system is admissibility which includes stability, regularity and causality for a discrete-time system (or impulse elimination for a continuous-time system). Consequently, over past decades, many different practical systems and research areas⁽¹⁻⁵⁾ have been adopted the singular system description. Many

fundamentals and advanced results for a normal state-space system have been successfully extended to a singular system, such as controllability and observability⁽⁶⁾, (robust) stability analysis and (robust) stabilization⁽⁷⁻⁹⁾, H_∞ control and filtering⁽¹⁰⁻¹²⁾ and so forth. In addition, in order to get less conservative results, singular system based methods have been adopted to study state-space systems^(13,14).

On the other hand, when a dynamic system experiences sudden change in its structure or parameters, it can be modelled as a Markovian jumping system (MJS). As a class of important stochastic hybrid systems, MJSs have attracted extensive and sustained research attention in control and filtering problems due to their theoretical importance and wide practical applications in manufacturing systems, economic systems, power systems, aerospace systems, and networked control systems^(15,16). Following the researching trends of a state-space system, in recent years, more notions and results in a normal MJS have been successfully extended to a singular Markovian jumping system, for example, stability and stabilization^(17,18), H_∞ control and filtering problems^(18,19) and so on. It should be pointed out that a discrete-time Markovian jumping singular system is more important than its continuous-time counterpart in our digital world.

For control systems, including singular and Markovian jumping systems, it has been recognized that the state variables are not often available or measurable due to technical or economical constraints in most practical situations. Therefore, a controller design that does not require complete access to the state vector is desirable. That is why observer-based control problems and observer design problems, which are involved with using the available information on inputs and outputs to reconstruct the unmeasured states of the concerned system, have been wide

investigated in many practical applications⁽²⁰⁻²²⁾. However, only a few works have been carried out on singular systems^(23,24), not to mention on the ones simultaneously considering Markovian jumping cases^(25,26). Up to date and to the best of our knowledge, the observer-based control problem for a discrete-time Markovian jumping singular system (DMJSS) does not have clear investigation and still remains challenges. This observation motivates our studies. In this paper, we extend our recent results⁽²⁷⁾ consider a mode dependent observer and controller co-design problem for a DMJSS with polytopic uncertainties. The contributions of this paper are as follows. (a) We generalize problem formulation to a system with polytopic uncertainties and extend the resultant LMI-based conditions⁽²⁷⁾ for an admissibility analysis problem. (b) We unify a method to deal with two design tasks, and derive two sets of strictly LMI-based synthesis conditions for an observer and a controller.

The rest of this paper is organized as follows. Section 2 presents some preliminaries and problem description. Section 3 gives the main results. Section 4 provides a numerical example to show the feasibility of the proposed method, and Section 5 concludes this paper. Throughout this paper, the following notations are adopted. Matrices are assumed to have compatible dimensions for algebraic operations if their dimensions are not explicitly stated. For any matrix Φ , $\text{sym}(\Phi) = \Phi + \Phi^T$, T is a transpose of a matrix. $\mathbf{P} > \mathbf{0}$ ($< \mathbf{0}$) means that the symmetric matrix $\mathbf{P}$ is positive definite (negative definite). $\mathbb{Z}$ represents the set of non-negative integer numbers, and $\text{diag}\{\cdot\}$ denotes a block diagonal matrix. $\mathbf{I}$ and $\mathbf{0}$ are identity and zero matrices, respectively, with appropriate dimensions.

2. Preliminaries and Problem Formulation

Consider an uncertain discrete-time Markovian jumping singular system (DMJSS), whose plant is

$$\Sigma_s : \begin{cases} \mathbf{E}\mathbf{x}(k+1) = \mathbf{A}_{\theta(k)}^\delta \mathbf{x}(k) + \mathbf{B}_{\theta(k)}^\delta \mathbf{u}(k), \\ \mathbf{y}(k) = \mathbf{C}_{\theta(k)}^\delta \mathbf{x}(k), \end{cases} \quad (1)$$

where $k \in \mathbb{Z}$, $\mathbf{x}(k) \in \mathbb{R}^n$ is the state vector, $\mathbf{u}(k) \in \mathbb{R}^p$ is the input vector, and $\mathbf{y}(k) \in \mathbb{R}^q$ is the measured output vector. $\{\theta(k), k \in \mathbb{Z}\}$ is a discrete-time, discrete-state Markovian chain taking values in a finite set $S = \{1, 2, \dots, \mathcal{N}\}$, with the following given transition probability from mode i at time k to mode j at time $k+1$, $k \in \mathbb{Z}$:

$$P_r[\theta(k+1) = j | \theta(k) = i] = p_{ij} \quad (2)$$

for $i, j \in S$, and $\sum_{j=1}^{\mathcal{N}} p_{ij} = 1$. The matrix $\mathbf{E} \in \mathbb{R}^{n \times n}$ is singular with $\text{rank}(\mathbf{E}) = r < n$. Other system matrices are supposed to be uncertain but belong to a known convex compact set of polytopic type:

$$\psi_{\theta(k)} = (\mathbf{A}_{\theta(k)}^\delta, \mathbf{B}_{\theta(k)}^\delta, \mathbf{C}_{\theta(k)}^\delta) \in \varphi_{\theta(k)}, \quad (3)$$

where

$$\varphi_{\theta(k)} = \left\{ \psi_{\theta(k)} \mid \psi_{\theta(k)} = \sum_{l=1}^v \beta_l \psi_{\theta(k)}^l, \sum_{l=1}^v \beta_l = 1, \beta_l \geq 0 \right\}, \quad (4)$$

and $\psi_{\theta(k)}^l = (\mathbf{A}_{\theta(k)}^l, \mathbf{B}_{\theta(k)}^l, \mathbf{C}_{\theta(k)}^l)$ is the l^{th} vertex of the polyhedral domain $\varphi_{\theta(k)}$, where $l \in Y = \{1, 2, \dots, v\}$. For notational simplicity, in the sequel, for each $\theta(k) = i$, $i \in S$, the matrices $\mathbf{A}_{\theta(k)}^l, \mathbf{B}_{\theta(k)}^l, \mathbf{C}_{\theta(k)}^l$ will be denoted by $\mathbf{A}_i^l, \mathbf{B}_i^l, \mathbf{C}_i^l$ respectively, where $\mathbf{A}_i^l, \mathbf{B}_i^l, \mathbf{C}_i^l$ are known constant matrices with appropriate dimensions.

Similar to the definition⁽⁷⁾ related to the admissibility for an unforced DMJSS, we can define the same admissibility for an uncertain unforced DMJSS in the following.

Definition 1. The unforced DMJSS Σ_s for all vertexes in $\varphi_{\theta(k)}$ (simply denotes as $(\mathbf{E}, \mathbf{A}_i^l)$) is said to be

- (a) **regular**, if for each $i \in S$, $l \in Y$, $\det(z\mathbf{E} - \mathbf{A}_i^l)$ is not identically zero;
- (b) **causal**, if for each $i \in S$, $l \in Y$, $\deg(\det(z\mathbf{E} - \mathbf{A}_i^l)) = \text{rank}(\mathbf{E})$;
- (c) **stochastically stable**, if for any $\mathbf{x}(0)$ and $\theta(0) \in S$, there exists a scalar $\Omega(\mathbf{x}(0), \theta(0))$ such that

$$\mathcal{E} \left\{ \sum_{k=0}^{\infty} \|\mathbf{x}(k, \mathbf{x}(0), \theta(0))\|^2 \mid \mathbf{x}(0), \theta(0) \right\} < \Omega(\mathbf{x}(0), \theta(0)),$$

where $\mathbf{x}(k, \mathbf{x}(0), \theta(0))$ is the solution to the unforced system Σ_s at time k under initial conditions $\mathbf{x}(0)$ and $\theta(0)$.

- (d) **stochastically admissible**, if it is regular, causal, and stochastically stable.

Similar to the Definition 2.1⁽²⁸⁾ related to R-controllable and controllable (R-observable and observable) for a discrete-time singular system, we can define the same controllability/observability for DMJSS in (1), and suppose the system Σ_s is controllable and observable.

Definition 2. The system Σ_s in (1) is said to be

- (a) **R-controllable**, if $\text{rank}[z\mathbf{E} - \mathbf{A}_i^l \quad \mathbf{B}_i^l] = n$;
- (b) **controllable**, if it is both R-controllable and $\text{rank}[\mathbf{E} \quad \mathbf{B}_i^l] = n$;
- (c) **R-observable**, if $\text{rank} \begin{bmatrix} z\mathbf{E} - \mathbf{A}_i^l \\ \mathbf{C}_i^l \end{bmatrix} = n$;
- (d) **observable**, if it is both R-observable and

$\text{rank} \begin{bmatrix} \mathbf{E} \\ \mathbf{C}_i^l \end{bmatrix} = n$, for all $i \in S, l \in \Upsilon, z \in \mathbb{C}$, z is finite.

Now, consider a Luenberger observer

$$\begin{aligned} \mathbf{E}\hat{\mathbf{x}}(k+1) &= \mathbf{A}_{\theta(k)}^\delta \hat{\mathbf{x}}(k) + \mathbf{B}_{\theta(k)}^\delta \mathbf{u}(k) \\ &+ \mathbf{O}_{\theta(k)} (\mathbf{y}(k) - \mathbf{C}_{\theta(k)}^\delta \hat{\mathbf{x}}(k)), \end{aligned} \quad (5)$$

and a control law

$$\mathbf{u}(k) = \mathbf{K}_{\theta(k)} \hat{\mathbf{x}}(k), \quad (6)$$

for Σ_s in (1), where $\mathbf{O}_{\theta(k)}$ and $\mathbf{K}_{\theta(k)}$ are observer gains and controller gains in connection with different operation modes to be designed. The corresponding augmented closed-loop dynamics Σ_a can be formulated as

$$\Sigma_a : \hat{\mathbf{E}}\zeta(k+1) = \hat{\mathbf{A}}_{\theta(k)}^\delta \zeta(k), \quad (7)$$

where $\zeta(k) = [\mathbf{x}^T(k) \quad \mathbf{e}^T(k)]^T$, $\mathbf{e}(k) = \mathbf{x}(k) - \hat{\mathbf{x}}(k)$, $\hat{\mathbf{E}} = \text{diag}\{\mathbf{E}, \mathbf{E}\}$ and

$$\hat{\mathbf{A}}_{\theta(k)}^\delta = \begin{bmatrix} \mathbf{A}_{\theta(k)}^\delta + \mathbf{B}_{\theta(k)}^\delta \mathbf{K}_{\theta(k)} & -\mathbf{B}_{\theta(k)}^\delta \mathbf{K}_{\theta(k)} \\ \mathbf{0} & \mathbf{A}_{\theta(k)}^\delta - \mathbf{O}_{\theta(k)} \mathbf{C}_{\theta(k)}^\delta \end{bmatrix}. \quad (8)$$

The purpose of this paper is to design a set of suitable observer gains and controller gains in (5) and (6), so that the augmented closed-loop system in (7)-(8) is stochastically admissible. In the following, we also denote $\mathbf{O}_{\theta(k)}$, $\mathbf{K}_{\theta(k)}$ as $\mathbf{O}_i$ and $\mathbf{K}_i$ to simplify notations for each $\theta(k) = i$, $i \in S$.

For our analysis and design tasks, the following three lemmas are required. Lemma 1 which is an extension of Lemma 3⁽²⁷⁾ is essential for our resultant conditions related to the corresponding analysis problems. Lemma 2 which is an extension of Lemma 4⁽²⁷⁾ enables us to consider the observer design and the controller design problems independently. Lemma 3 provides us a key idea so that we can unify our methods for the design tasks.

Lemma 1. The unforced uncertain DMJSS $(\mathbf{E}, \mathbf{A}_i^l)$ is stochastically admissible for all $i \in S$ if and only if there are feasible matrices $\mathbf{Y}_i$ and positive definite matrices $\mathbf{Z}_i$, $\mathbf{\Lambda}_i$ satisfying the strict LMIs in (9).

$$\begin{bmatrix} \mathbf{\Pi}_{1i}^l & \mathbf{\Pi}_{2i}^{lT} \\ \mathbf{\Pi}_{2i}^l & \mathbf{\Pi}_3 \end{bmatrix} < \mathbf{0}, \quad l \in \Upsilon, i \in S, \quad (9)$$

where

$$\begin{aligned} \mathbf{\Pi}_{1i}^l &= \text{sym} \left(\left(\sqrt{p_{ii}} \mathbf{A}_i^l - \mathbf{E} \right) \left(\mathbf{Z}_i \mathbf{E}^T + \mathbf{L} \mathbf{Y}_i \right) \right), \\ \mathbf{\Pi}_{2i}^l &= \left(\mathbf{p}_i^T \mathbf{A}_i^l - \mathbf{I}_{(i)}^T \mathbf{E} \right) \left(\mathbf{Z}_i \mathbf{E}^T + \mathbf{L} \mathbf{Y}_i \right), \\ \mathbf{\Pi}_3 &= -\text{diag} \left\{ \mathbf{E} \mathbf{Z}_1 \mathbf{E}^T + \mathbf{U} \mathbf{\Lambda}_1 \mathbf{U}^T, \dots, \mathbf{E} \mathbf{Z}_N \mathbf{E}^T + \mathbf{U} \mathbf{\Lambda}_N \mathbf{U}^T \right\}, \\ \mathbf{p}_i &= \left[\sqrt{p_{i1}} \mathbf{I} \cdots \sqrt{p_{iN}} \mathbf{I} \right], \quad \mathbf{I}_{(i)} = \begin{bmatrix} \mathbf{0} & \cdots & \mathbf{I} & \cdots & \mathbf{0} \end{bmatrix}_{i-th}, \end{aligned} \quad (10)$$

and matrix $\mathbf{L}$ is a given full-column-rank matrix satisfying $\mathbf{E} \mathbf{L} = \mathbf{0}$; $\mathbf{U}^T = [\mathbf{0} \quad \mathbf{I}_{n-r}] \mathbf{M}_0^{-T}$ where $\mathbf{M}_0$ is nonsingular satisfying $\mathbf{M}_0 \mathbf{E} \mathbf{N}_0 = \text{diag}\{\mathbf{I}_r, \mathbf{0}\}$.

Lemma 2. Suppose both singular systems $(\mathbf{E}, \mathbf{A}_K)$ and $(\mathbf{E}, \mathbf{A}_O)$ are admissible, where $\mathbf{E}, \mathbf{A}_K, \mathbf{A}_O \in \mathbb{R}^{n \times n}$ and $\text{rank}(\mathbf{E}) = r < n$. Then the augmented singular system $(\bar{\mathbf{E}}, \bar{\mathbf{A}})$ is admissible, where

$$(\bar{\mathbf{E}}, \bar{\mathbf{A}}) = \left(\begin{bmatrix} \mathbf{E} & \mathbf{0} \\ \mathbf{0} & \mathbf{E} \end{bmatrix}, \begin{bmatrix} \mathbf{A}_K & \mathbf{B}_K \\ \mathbf{0} & \mathbf{A}_O \end{bmatrix} \right),$$

and $\mathbf{B}_K$ is a parameter matrix for the augmented system.

Lemma 3. Suppose a discrete-time singular system $(\mathbf{E}, \mathbf{A}_O)$ is regular and causal, where $\mathbf{E}, \mathbf{A}_O \in \mathbb{R}^{n \times n}$, $\text{rank}(\mathbf{E}) = r < n$. Then these two discrete-time singular systems $(\mathbf{E}^T, \mathbf{A}_O^T)$ and $(\mathbf{E}, \mathbf{A}_O)$ have same admissibility.

Proof. It is obvious that these two systems are restricted system equivalence⁽¹⁾. Lemma 3 is straightforward according to the property that singular systems with restricted system equivalence have same admissibility⁽²⁹⁾.

3. Main Results

3.1 Stochastic Admissibility Analysis

Along with Lemma 1 and the convex properties of polyhedral domain, we can easily modified Lemma 1 to be Theorem 1 which could suitably apply to analyze the augmented system Σ_a with polytopic uncertainties.

Theorem 1. For a set of given $\mathbf{O}_i$ and $\mathbf{K}_i$ ($i \in S$), the uncertain DMJSS Σ_a (simply denotes as $(\bar{\mathbf{E}}, \bar{\mathbf{A}}_i^l)$) is stochastically admissible if and only if there exist feasible matrices $\hat{\mathbf{Y}}_i$ and positive definite matrices $\hat{\mathbf{Z}}_i$, $\hat{\mathbf{\Lambda}}_i$ satisfying LMIs (11).

$$\begin{bmatrix} \hat{\mathbf{\Pi}}_{1i}^l & \left(\hat{\mathbf{\Pi}}_{2i}^l \right)^T \\ \hat{\mathbf{\Pi}}_{2i}^l & \hat{\mathbf{\Pi}}_3 \end{bmatrix} < \mathbf{0}, \quad l \in \Upsilon, i \in S, \quad (11)$$

where

$$\begin{aligned}\hat{\Pi}_{li}^l &= \text{sym}\left(\left(\sqrt{p_{ii}}\hat{\mathbf{A}}_i^l - \hat{\mathbf{E}}\right)\left(\hat{\mathbf{Z}}_i\hat{\mathbf{E}}^T + \hat{\mathbf{L}}\hat{\mathbf{Y}}_i\right)\right), \\ \hat{\Pi}_{2i}^l &= \left(\mathbf{p}_i^T\hat{\mathbf{A}}_i^l - \mathbf{I}_{(i)}^T\hat{\mathbf{E}}\right)\left(\hat{\mathbf{Z}}_i\hat{\mathbf{E}}^T + \hat{\mathbf{L}}\hat{\mathbf{Y}}_i\right), \\ \hat{\Pi}_3 &= -\text{diag}\left\{\hat{\mathbf{E}}\hat{\mathbf{Z}}_1\hat{\mathbf{E}}^T + \hat{\mathbf{U}}\hat{\Lambda}_1\hat{\mathbf{U}}^T, \dots, \hat{\mathbf{E}}\hat{\mathbf{Z}}_N\hat{\mathbf{E}}^T + \hat{\mathbf{U}}\hat{\Lambda}_N\hat{\mathbf{U}}^T\right\},\end{aligned}\quad (12)$$

matrices $\mathbf{p}_i$ and $\mathbf{I}_{(i)}$ are defined in (10), and $\hat{\mathbf{L}}$ is a given full-column-rank matrix satisfying $\hat{\mathbf{E}}\hat{\mathbf{L}} = \mathbf{0}$; $\hat{\mathbf{U}}^T = [\mathbf{0} \ \mathbf{I}_{2(n-r)}]\hat{\mathbf{M}}_0^{-T}$ where $\hat{\mathbf{M}}_0$ is nonsingular satisfying $\hat{\mathbf{M}}_0\hat{\mathbf{E}}\hat{\mathbf{N}}_0 = \text{diag}\{\mathbf{I}_{2r}, \mathbf{0}\}$.

Remark 1. In accordance with Lemma 2, the admissibility of the augmented system Σ_a depends on the admissibility of two subsystems $(\mathbf{E}, \mathbf{A}_i^l + \mathbf{B}_i^l\mathbf{K}_i)$ and $(\mathbf{E}, \mathbf{A}_i^l - \mathbf{O}_i\mathbf{C}_i^l)$. We can also separately analyze these two subsystems instead.

3.2 Controller Design and Observer Design

According to Lemma 2, we can separately consider two subsystems $(\mathbf{E}, \mathbf{A}_i^l + \mathbf{B}_i^l\mathbf{K}_i)$ and $(\mathbf{E}, \mathbf{A}_i^l - \mathbf{O}_i\mathbf{C}_i^l)$ in (7)-(8) to design a set of feasible observer gains and controller gains instead of using augmented system Σ_a directly. In the following, Theorem 2 and Theorem 3 respectively provide a set of necessary and sufficient strictly LMI-based conditions for the controller design and observer design problems.

Theorem 2. (Controller design) The uncertain subsystem $(\mathbf{E}, \mathbf{A}_i^l + \mathbf{B}_i^l\mathbf{K}_i)$ is stochastically admissible if and only if there exist feasible matrices Ω_i , $\mathbf{Y}_i$, and positive definite matrices $\mathbf{Z}_i$, Λ_i for $i \in S$ satisfying LMIs (13).

$$\begin{bmatrix} \Psi_{li}^l & (\Psi_{2i}^l)^T \\ \Psi_{2i}^l & \Psi_3 \end{bmatrix} < \mathbf{0}, \quad i \in S, l \in \Upsilon, \quad (13)$$

where

$$\begin{aligned}\Psi_{li}^l &= \text{sym}\left(\left(\sqrt{p_{ii}}\mathbf{A}_i^l - \mathbf{E}\right)\mathbf{W}_i + \sqrt{p_{ii}}\mathbf{B}_i^l\Omega_i\right), \\ \Psi_{2i}^l &= \mathbf{p}_i^T\left(\mathbf{A}_i^l\mathbf{W}_i + \mathbf{B}_i^l\Omega_i\right) - \mathbf{I}_{(i)}^T\mathbf{E}\mathbf{W}_i, \\ \Psi_3 &= -\text{diag}\left\{\mathbf{E}\mathbf{Z}_1\mathbf{E}^T + \mathbf{U}\Lambda_1\mathbf{U}^T, \dots, \mathbf{E}\mathbf{Z}_N\mathbf{E}^T + \mathbf{U}\Lambda_N\mathbf{U}^T\right\},\end{aligned}\quad (14)$$

$\mathbf{W}_i = \mathbf{Z}_i\mathbf{E}^T + \mathbf{L}\mathbf{Y}_i$, and matrices $\mathbf{L}$, $\mathbf{U}$, $\mathbf{I}_{(i)}$, $\mathbf{p}_i$ are defined in Lemma 1. If in this case, the controller gains in (6) can be found by

$$\mathbf{K}_i = \Omega_i\mathbf{W}_i^{-1}. \quad (15)$$

Sketch of Proof. By Theorem 1, we can easily get (13)-(14) when we replace $\hat{\mathbf{E}}$, $\hat{\mathbf{A}}_i^l$ with $\mathbf{E}$, $(\mathbf{A}_i^l + \mathbf{B}_i^l\mathbf{K}_i)$, respectively, and define $\Omega_i = \mathbf{K}_i(\mathbf{Z}_i\mathbf{E}^T + \mathbf{L}\mathbf{Y}_i)$.

By Lemma 3, it is evident that $(\mathbf{E}, \mathbf{A}_i^l - \mathbf{O}_i\mathbf{C}_i^l)$ and $(\mathbf{E}^T, \mathbf{A}_i^{lT} - \mathbf{C}_i^{lT}\mathbf{O}_i^T)$ have same admissibility. In order to design feasible observer gains so that $(\mathbf{E}, \mathbf{A}_i^l - \mathbf{O}_i\mathbf{C}_i^l)$ is

stochastically admissible, we can consider to design the system $(\mathbf{E}^T, \mathbf{A}_i^{lT} - \mathbf{C}_i^{lT}\mathbf{O}_i^T)$ instead.

Theorem 3. (Observer design) The uncertain error dynamics $(\mathbf{E}, \mathbf{A}_i^l - \mathbf{O}_i\mathbf{C}_i^l)$ which is a subsystem of Σ_a is stochastically admissible for all $i \in S$ if and only if there exist matrices $\bar{\Omega}_i$, $\bar{\mathbf{Y}}_i$, and positive definite matrices $\bar{\mathbf{Z}}_i$, $\bar{\Lambda}_i$, such that LMIs (16) hold.

$$\begin{bmatrix} \Phi_{li}^l & (\Phi_{2i}^l)^T \\ \Phi_{2i}^l & \Phi_3 \end{bmatrix} < \mathbf{0}, \quad i \in S, l \in \Upsilon, \quad (16)$$

where

$$\begin{aligned}\Phi_{li}^l &= \text{sym}\left(\left(\sqrt{p_{ii}}\mathbf{A}_i^{lT} - \mathbf{E}^T\right)\bar{\mathbf{W}}_i - \sqrt{p_{ii}}\mathbf{C}_i^{lT}\bar{\Omega}_i\right), \\ \Phi_{2i}^l &= \mathbf{p}_i^T\left(\mathbf{A}_i^{lT}\bar{\mathbf{W}}_i - \mathbf{C}_i^{lT}\bar{\Omega}_i\right) - \mathbf{I}_{(i)}^T\mathbf{E}^T\bar{\mathbf{W}}_i, \\ \Phi_3 &= -\text{diag}\left\{\mathbf{E}^T\bar{\mathbf{Z}}_1\mathbf{E} + \bar{\mathbf{U}}^T\bar{\Lambda}_1\bar{\mathbf{U}}, \dots, \mathbf{E}^T\bar{\mathbf{Z}}_N\mathbf{E} + \bar{\mathbf{U}}^T\bar{\Lambda}_N\bar{\mathbf{U}}\right\},\end{aligned}\quad (17)$$

$\bar{\mathbf{W}}_i = \bar{\mathbf{Z}}_i\mathbf{E} + \bar{\mathbf{L}}\bar{\mathbf{Y}}_i$, and matrix $\bar{\mathbf{L}}$ is a given full-column-rank matrix satisfying $\mathbf{E}^T\bar{\mathbf{L}} = \mathbf{0}$; $\bar{\mathbf{U}} = [\mathbf{0} \ \mathbf{I}_{n-r}]\mathbf{M}_0^{-T}$ where $\mathbf{M}_0$ is nonsingular satisfying $\mathbf{M}_0\mathbf{E}\mathbf{N}_0 = \text{diag}\{\mathbf{I}_r, \mathbf{0}\}$. If in this case, the observer gains in (5) can be found by

$$\mathbf{O}_i = \bar{\mathbf{W}}_i^{-T}\bar{\Omega}_i^T. \quad (18)$$

Sketch of Proof. We can easily derive (16)-(17) from (13)-(14) via replacing the quadruple $(\mathbf{E}, \mathbf{A}_i^l, \mathbf{B}_i^l, \mathbf{K}_i)$ with $(\mathbf{E}^T, \mathbf{A}_i^{lT}, -\mathbf{C}_i^{lT}, \mathbf{O}_i^T)$ and modifying related variables and parameter matrices.

Remark 2. Comparing with our recent results⁽²⁷⁾, there are two advantages and differences in this paper. One is that the assumption that $\mathbf{C}$ must be full-row-rank is unnecessary and is removed in this paper, which fits more practical cases. Another is that the resultant conditions are extended to be feasible for uncertain systems. In addition, we unify one method for these two design tasks.

4. A Numerical Example

Consider a two-mode jumping system (1) with the following matrices:

$$\begin{aligned}\text{Mode 1: } [\mathbf{A}_1 \mid \mathbf{B}_1 \mid \mathbf{C}_1^T] &= \begin{bmatrix} -1 & 0 & 0.2 & 0 & -1 & 0.1 \\ 0 & 3 & \alpha & -1 & -1 & -0.2 \\ 1 & 1 & 1 & -1 & -1 & 0.1 \end{bmatrix}, \\ \text{Mode 2: } [\mathbf{A}_2 \mid \mathbf{B}_2 \mid \mathbf{C}_2^T] &= \begin{bmatrix} 1 & 1 & -0.8 & 0 & -3 & 0.2 \\ 0 & 2 & \beta & 6 & 3 & 0.2 \\ 1 & 1.5 & 1 & 1.5 & -3 & 0.2 \end{bmatrix}, \\ \mathbf{E} &= \begin{bmatrix} 1 & 0 & 1 \\ 3 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \begin{matrix} -0.2 \leq \alpha \leq 2, \\ -3 \leq \beta \leq -1.8. \end{matrix}\end{aligned}$$

There are two vertexes in every operation mode for this system. The transition probability matrix is supposed to be

$$[p_{ij}] = \begin{bmatrix} 0.6 & 0.4 \\ 0.3 & 0.7 \end{bmatrix}, \quad \{i, j\} \in S = \{1, 2\}.$$

Obviously, $\text{rank}(\mathbf{E}) = 2$. The jumping system is observable and controllable according to Definition 2, but it is not stochastically admissible because there is no feasible solutions $(\mathbf{Y}_i, \mathbf{Z}_i, \mathbf{\Lambda}_i)$ for the conditions (9)-(10) in Lemma 1. In order to design the observer in (5) and the controller in (6) so that the uncertain closed-loop system Σ_a in (7)-(8) is stochastically admissible for all jumping modes, we apply the resultant LMI conditions in Theorem 3 and Theorem 2 to be feasibility problems⁽³⁰⁾, respectively.

For the observer design problem, via Theorem 3, we get the following feasible solutions.

$$\begin{bmatrix} \bar{\mathbf{Y}}_1 \\ \bar{\mathbf{Y}}_2 \end{bmatrix} = \begin{bmatrix} -0.3514 & -0.0540 & 0.1624 \\ -0.4621 & -0.1497 & 0.5527 \end{bmatrix}, \begin{bmatrix} \bar{\mathbf{\Lambda}}_1 \\ \bar{\mathbf{\Lambda}}_2 \end{bmatrix} = \begin{bmatrix} 1.2105 \\ 1.2715 \end{bmatrix},$$

$$\begin{bmatrix} \bar{\mathbf{Z}}_1 \\ \bar{\mathbf{Z}}_2 \end{bmatrix} = \begin{bmatrix} 0.4750 & -0.1130 & 0 \\ -0.1130 & 0.0952 & 0 \\ 0 & 0 & 1.3307 \\ 0.4731 & -0.1132 & 0 \\ -0.1132 & 0.1025 & 0 \\ 0 & 0 & 1.3307 \end{bmatrix}, \begin{bmatrix} \bar{\mathbf{\Omega}}_1^T \\ \bar{\mathbf{\Omega}}_2^T \end{bmatrix} = \begin{bmatrix} -1.8329 \\ -2.9074 \\ -0.1914 \\ -3.0610 \\ 0.4289 \\ 3.9989 \end{bmatrix}.$$

And the corresponding feasible observer gains in (5) are

$$[\mathbf{O}_1 \mid \mathbf{O}_2] = \begin{bmatrix} -4.8930 & -3.5717 \\ -47.7660 & 21.7694 \\ -20.1197 & 14.7510 \end{bmatrix}.$$

For the controller design problem, via Theorem 2, we have the following feasible solutions.

$$\begin{bmatrix} \mathbf{Y}_1 \\ \mathbf{Y}_2 \end{bmatrix} = \begin{bmatrix} -0.9117 & -4.7385 & 20.3433 \\ -12.9313 & -8.2422 & 4.8040 \end{bmatrix}, \begin{bmatrix} \mathbf{\Lambda}_1 \\ \mathbf{\Lambda}_2 \end{bmatrix} = \begin{bmatrix} 103.2705 \\ 103.5968 \end{bmatrix},$$

$$\begin{bmatrix} \mathbf{\Omega}_1 \\ \mathbf{\Omega}_2 \end{bmatrix} = \begin{bmatrix} 16.3352 & 104.5803 & 28.3208 \\ -13.5404 & -58.6258 & 16.0706 \\ 31.1795 & -12.3260 & -5.6081 \\ -12.0413 & 18.9738 & 7.0627 \end{bmatrix},$$

$$\begin{bmatrix} \mathbf{Z}_1 \\ \mathbf{Z}_2 \end{bmatrix} = \begin{bmatrix} 15.5397 & -17.3955 & -2.1623 \\ -17.3955 & 87.4299 & 13.7694 \\ -2.1623 & 13.7694 & 24.1566 \\ 30.4918 & -61.5914 & 3.0652 \\ -61.5914 & 230.9701 & -6.3493 \\ 3.0652 & -6.3493 & 12.1437 \end{bmatrix}.$$

And the corresponding feasible controller gains in (6) are

$$\begin{bmatrix} \mathbf{K}_1 \\ \mathbf{K}_2 \end{bmatrix} = \begin{bmatrix} -0.1599 & 2.9994 & 1.3921 \\ -2.2237 & 0.1234 & 0.7900 \\ 0.0832 & -0.4570 & -1.1674 \\ 0.3120 & 0.3806 & 1.4702 \end{bmatrix}.$$

According to the given transition probability matrix, the jumping modes are presented in Figure 1. Taking these feasible gains to the closed-loop system in (7)-(8) under the operating situation (shown in Figure 1), the simulation results of the augmented system's states $(\mathbf{e}(k), \mathbf{x}(k))$ for different (α, β) pairs are shown in Figure 2 and Figure 3, respectively. Both Figure 2 and Figure 3 illustrate the stability of the controlled system and the efficiency of the proposed resultant conditions. In addition, Figure 2 and Figure 3 also show that uncertainties affect $e_3(k)$ and $x_3(k)$.

Fig. 1 Operating modes

Fig. 2 Error responses for different (α, β)

Fig. 3 States responses for different (α, β)

5. Conclusions

In this paper, we investigate an observer-based control problem for a discrete-time Markovian jumping singular system with polytopic uncertainties. We not only generalize the problem formulation to a system with polytopic uncertainties, but also extend our recent resultant LMI-based conditions for an admissibility analysis problem. In addition, we propose a method to separately deal with the observer and controller design problems, and derive two sets of strictly LMI-based synthesis conditions. The simulation results of a numerical example show that the proposed conditions are feasible and efficient.

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