Dependence of CGH reconstruction due to the transfer phase function of angular spectrum method

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Abstract

Computer-generated hologram (CGH) is the method of making holograms with computer. A CGH can be made without preparing and instructing the exclusive optical tool. Therefore, we can make holograms easily and economically compared to the optical method.

There are several calculating methods for a CGH such as Fourier transform, Fresnel diffraction and etc. In this study, we discuss on the angular spectrum method which is not limited by the diffraction distance. The Kirchhoff integral has the advantage of making reconstructions similar to optical experiment. However, the angular spectrum method has the disadvantage of accruing an error without proper conditions. These conditions are related to the transfer phase function. Moreover, the transfer phase function is related to the sampling interval and the number of sampling points. Therefore, it is necessary to investigate the relationship about these conditions and reconstruction errors.

This study reports on the reconstruction using the angular spectrum method under different conditions about the transfer phase function. Simulation results will be shown on the relationship under these conditions.

Keywords: Computer-generated hologram, optical diffraction, Fourier transform, angular spectrum method.

1. Angular spectrum method

1.1 Transfer phase function

Angular spectrum method uses discrete Fourier transform. In this method, hologram is made as follows. First, original image is discrete Fourier transform. Next, multiplying the transfer phase function and this image. Lastly, this image is inverse discrete Fourier transform.

The formula about transfer phase function is written as (1).

$$\exp \left\{ j k z \sqrt{1 - (\lambda f_x)^2 - (\lambda f_y)^2} \right\} \quad (1)$$

In this formula, z is distance between the original image and the hologram. k is wavenumber. λ is wavelength. f_x and f_y are spatial frequency. The spatial frequency is inverse proportion to the sampling interval. Therefore, transfer phase function depends on z and the sampling interval. Formula (2) is complex amplitude distribution of hologram made of angular spectrum method.

$$U(f_x, f_y, z) = \mathcal{F}^{-1} \left[\mathcal{F}\{g(x, y, 0)\} \exp \left(j \frac{2\pi}{\lambda} z \sqrt{1 - (\lambda f_x)^2 - (\lambda f_y)^2} \right) \right] \quad (2)$$

1.2 Discretization

As mentioned above, angular spectrum method uses discrete Fourier transform. Therefore, it is need to consider discretization. Formula (3) is Fourier transform about original image. Discretizing formula (3) to calculate formula (4).

$$G(f) = \iint_{-\infty}^{\infty} g(x) \exp(-j2\pi f x) dx \quad (3)$$

$$k \Delta f \cdot l \Delta x = \frac{k l}{N} \quad (4)$$

In formula (4), N is a number of sampling points. As with original image, discretizing hologram. As a result, formula (5) is got.

$$m \Delta f' \cdot n \Delta u = \frac{m n}{N'} \quad (5)$$

Formula (6) is consisted by formula (4) and (5).

$$\Delta x N = \Delta u N' \quad (6)$$

Usually, N is equal to N' in angular spectrum method. Therefore, Formula (7) is established.

$$\Delta x = \Delta u \quad (7)$$

In formula (7), Δx is sampling interval of original image. Δu is sampling interval of hologram. Formula (7) shows that sampling interval of original image and hologram are need to be equal.

Formula (8) is discretize the complex amplitude distribution of the hologram made of angular spectrum method.

$$U(k\Delta f_x, l\Delta f_y, z) = \mathcal{F}^{-1} \left[\frac{\mathcal{F}\{g(m\Delta x, n\Delta y, 0)\}}{\exp\left(j\frac{2\pi}{\lambda}z\sqrt{1-\left(\frac{\lambda}{m\Delta x N}\right)^2-\left(\frac{\lambda}{n\Delta y N}\right)^2}\right)} \right] \quad (8)$$

$$\begin{aligned} k\Delta f_x &= f_x, l\Delta f_y = f_y \\ m\Delta x &= x, n\Delta y = y \end{aligned} \quad (9)$$

1.3 Comparing angular spectrum method and Kirchhoff integral

In general, the Kirchhoff integral is used to CGH. Because, this method make reconstructions similar to optical experiment. Therefore, in this section, compering reconstructions made of the Kirchhoff integral and angular spectrum method. Also, viewing the transfer phase function influences for angular spectrum method.

1.3.1 The error depending on distance

For formula (1), the transfer phase function depends on distance between the original image and the hologram. Therefore, examining how reconstructions change according to z . Fig. 1 is the original image. A number of sampling points of this image is 128×128 [pixel]. Sampling interval of this image is $20.0[\mu\text{m}]$. For formula (7), deciding that a number of sampling points of hologram is 128×128 [pixel] and sampling interval of hologram is $20.0[\mu\text{m}]$.

Making reconstructions of the Kirchhoff integral and angular spectrum method in the above conditions. Setting z to 0.1, 0.2, 0.4, 0.8[m]. Fig. 2 (a) ~ (d) are reconstructions made of the Kirchhoff integral. Fig. 3 (a) ~ (d) are reconstructions made of angular spectrum method. Moreover, Fig. 4 is a graph of the error of reconstructions in decibel notation. Formula (10) is used to find the error.

$$10\log_{10} E \quad (10)$$

In formula (10), E is a variable which shows difference in amplitudes of the original image and the reconstruction. Therefore, the larger the absolute value of formula (10), the smaller the error.

Fig. 1. Original image

A number of sampling points: 128×128 [pixel]

Sampling interval: $20.0[\mu\text{m}]$

(a) Distance: 0.1[m]

(d) Distance: 0.2[m]

(c) Distance: 0.4[m]

(b) Distance: 0.8[m]

Fig. 2. Reconstructions made of the Kirchhoff integral

Fig. 3. Reconstructions made of angular spectrum method

Fig. 4. a graph of the error

For Fig.2 (a) ~ (d) and Fig.4, the larger z , the larger errors. Therefore, in general, errors of CGH reconstruction increases as the distance between the original image and the hologram. Moreover, for Fig.3 (a) ~ (d) and Fig.4, similar results can be obtained with angular spectrum method. From the above, if formula (7) holds, angular spectrum method will yields the same result as the Kirchhoff integral.

1.3.2 The error depending on sampling interval

Formula (11) is sampling theorem in making hologram.

$$\Delta u \leq \frac{\lambda z}{\Delta x N} \quad (11)$$

In formula (11), N is a number of sampling points of original image. Δx is sampling interval of original image. Δu is sampling interval of hologram. z is distance between the original image and the hologram. λ is wavelength. In this study, setting N to 128[pixel] and λ to 632[nm].

Making and compering reconstructions when formula (11) is satisfied, not satisfied, and the equation holds.

First, changing Δu only and making reconstructions using the Kirchhoff integral and angular spectrum method. In this time, setting Δx to 20.0[μm] and z to 0.1[m]. Therefore, if equation holds, Δu set to 24.7[μm]. Fig. 5 (a) ~ (e) are reconstructions made of the Kirchhoff integral. Fig. 6 is a graph of the error of reconstructions.

Fig. 5. Reconstructions made of the Kirchhoff integral

Fig. 6. error of reconstructions made of the Kirchhoff integral

For Fig. 5, Fig. 6, if equation holds, errors of reconstructions are the smallest. Moreover, if formula (11) is not satisfied, overlap occurs. Fig. 7 (a) ~ (d) are reconstructions made of angular spectrum method.

Fig. 7. Reconstructions made of angular spectrum method

For Fig. 7, if use angular spectrum method, the larger Δu , the bigger F letter.

Based on the above results, error of reconstructions made of the Kirchhoff integral is the smallest, when setting Δu to $24.7[\mu m]$. However, reconstructions of angular spectrum method have larger F letter than original image. Therefore, the Kirchhoff integral and angular spectrum method have different best conditions.

For Fig. 5(b), Fig. 7(b), when setting Δu to $24.7[\mu m]$, in other words, sampling interval of original image and

hologram are equal, there is little difference between the reconstruction and the original image. Therefore, when using angular spectrum method, sampling interval of original image and hologram must be equal.

In formula (11), when sampling interval of original image and hologram are equal, formula (12) is contracted.

$$\Delta u \leq \sqrt{\frac{\lambda z}{N}} \quad (12)$$

Making and compering reconstructions using the Kirchhoff integral when formula (12) is satisfied, not satisfied, and the equation holds. In this time, if equation holds, Δu set to $22.2[\mu m]$. Fig. 8 (a) ~ (d) are reconstructions made of the Kirchhoff integral. Fig. 9 (a) ~ (d) are reconstructions made of angular spectrum method. Moreover, Fig. 10 is a graph of the error of reconstructions.

Fig. 8. Reconstructions made of the Kirchhoff integral

Fig. 9. Reconstructions made of angular spectrum method

Fig. 10. error of reconstructions

For Fig. 10, as Δu approach 22.2[μm], errors of reconstruction made of each method decrease. Moreover, if formula (12) is not satisfied, overlap occurs. Therefore, when Δx and Δu are equal, reconstructions made of angular spectrum method and the Kirchhoff integral have the same tendency.

2. The relationship about transfer phase function and reconstruction

Fig. 11 (a) ~ (d) are graphs showing transfer phase function at each distance. For Fig 11, as distance increases, sampling interval of transfer phase function also increases. Therefore, information of hologram is missing and errors of reconstructions occur.

Fig.11. graphs showing transfer phase function at each distance

Conclusions

When making holograms using angular spectrum method, it is desirable to reduce interval of transfer phase function. However, if conditions such as sampling theorem are not taken into consideration, errors such as overlap will occur.

References

- (1) Joseph W. Goodman: Introduction to Fourier Optics, 2005
- (2) P.Harlharan: Basies of Holography, 2004