Application of Taguchi Experimental Method in Numerical Simulation of Variable Viscosity Effect on Natural Convection in Porous Media

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Abstract

The study uses an optimization approach representation for the variable viscosity effect on the free convection over a vertical permeable plate in porous media. The surface of the vertical plate is uniform wall temperature (UWT). The surface blowing/suction velocity is also uniform. The viscosity of the fluid varies inversely as a linear function of the temperature. The partial differential equations are transformed into non-similar equations and solved by Keller box method (KBM). Compared with the previously published articles, the results are considered to be very consistent. Numerical results for the the local Nusselt number with the two parameters: 1) blowing/suction parameter ξ , 2) viscosity-variation parameter θ_r are expressed in tables. Through the Taguchi method to predict the best point of the maxima of the local Nusselt number is 1.1152.

Keywords: Taguchi method, Variable viscosity, Free convection, Vertical permeable plate, Porous media.

1. Introduction

The heat transfer of free convection in saturated porous media has many important applications in nature and engineering. Examples include geothermal flow, nuclear waste storage, and electronic heat transfer systems.

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Minkowycz and Cheng⁽¹⁾ solved the uniform lateral mass flux on natural convection over a vertical plate in porous media by using the local non-similarity (LNS) approximation. The uniform blowing/suction effect on free convection past a vertical plate and cone with uniform wall

temperature (UWT) in porous media was solved by Yih⁽²⁾ utilizing Keller box method (KBM). He found that the local Nusselt number increase for the case of suction, but the blowing is the opposite.

The study of the variable viscosity in porous media has been widely discussed. Lai and Kulacki⁽³⁾ considered the effect of variable viscosity on convective heat transfer along a vertical surface in a saturated porous medium, and fluid viscosity has been assumed to be an inverse function of temperature. The effect of variable viscosity on flow and heat transfer characteristics for natural convection along an inclined plate in a saturated porous medium with an applied magnetic field has been explored by Tomer et al.⁽⁴⁾.

However, the Taguchi method to find the maximum values simulation of the research is still very few. Chou et al.⁽⁵⁾ studied application of Taguchi non-linear dynamic quality characteristics to fuzzy control system. Ho et al.⁽⁶⁾ reported adaptive network-based fuzzy inference system for prediction of surface roughness in end milling process using hybrid Taguchi-genetic learning algorithm. Taguchi optimization of bismuth-telluride based thermoelectric cooler was analyzed by Kishore et al.⁽⁷⁾.

In this exploring, the partial differential equations are transformed into non-similar equations and sloved in the Keller box method (KBM) proposed by Cebeci and Bradshaw⁽⁸⁾. The numerical simulation of two different parameters is carried out to obtain simulate the maximum value of the local Nusselt number and find the best parameter ratio. The study is divided into two phases: the first phase of the data is compared with the previously published article, the results are considered very consistent. The variable viscosity and blowing/suction effects on the local Nusselt number are presented in tabular and graphic forms. The second stage uses the Taguchi method to

optimize the two parameters by Chou⁽⁹⁾. We use the Taguchi experimental method to replace the single factor experiments to find the maximum local Nusselt number.

2. METHODS

2.1 Mathematical Equations

Phase 1: consider the influences of the variable viscosity and blowing/suction on the heat transfer by free convection flow over a vertical permeable flat plate embedded in a saturated porous medium. Figure 1 shows the concept map. The boundary condition is uniform wall temperature T_w (UWT).

Fig 1. Flow model and physical coordinate system.

The origin of the coordinate system is placed at the leading edge of the vertical flat plate, where x and y are Cartesian coordinates measuring distance along and normal to the surface of vertical flat plate, respectively. All the fluid properties are assumed to be constant, except for the viscosity and the density variation in the buoyancy term. The viscosity μ of the fluid-saturated porous medium depends on the temperature T in the following form (refer Lai and Kulacki⁽³⁾ and Vajravelu et al.⁽¹⁰⁾):

$$\frac{1}{\mu} = \frac{1}{\mu_\infty} [1 + \gamma(T - T_\infty)] = a(T - T_r) \quad (1)$$

Where γ is a viscosity-variation constant and μ_∞ is the viscosity of the ambient fluid with the following relation

$$a = \frac{\gamma}{\mu_\infty} \quad \text{and} \quad T_r = T_\infty - \frac{1}{\gamma} \quad (2)$$

Both a and T_r are constants and their values depend on the reference state and the thermal property of the fluid, i.e., γ . In general, $a > 0$ ($\gamma > 0$) for liquid and $a < 0$ ($\gamma < 0$) for gas. The viscosity of the fluid usually reduces with increasing temperature while it enhances for gas.

To further show the appropriateness of equation (1), correlations between viscosity and temperature for air and water are given below because these two are the most common working fluids found in engineering applications.

For air

$$\frac{1}{\mu} = -123.2(T - 742.6) \quad (3)$$

$T_\infty = 293\text{K}(20^\circ\text{C})$

and for water

$$\frac{1}{\mu} = 29.83(T - 258.6) \quad (4)$$

$T_\infty = 288\text{K}(15^\circ\text{C})$

The data used for these correlations are taken from Weast⁽¹¹⁾. While equation (3) is good to within 1.2% from 278K (5 °C) to 373 K (100 °C), equation (4) is good to within 5.8% from 283K (10 °C) to 373 K (100 °C). The reference temperatures thus selected for the correlations are very practical in most applications.

Introducing the boundary layer approximation and Boussinesq approximation, the governing equations and the boundary conditions based on the Darcy law can be written as follows:

Continuity equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (5)$$

Momentum (Darcy) equation:

$$u = -\frac{K}{\mu} \left(\frac{\partial p}{\partial x} + \rho g \right) \quad (6)$$

$$v = -\frac{K}{\mu} \cdot \frac{\partial p}{\partial y} \quad (7)$$

Energy equation:

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \cdot \frac{\partial^2 T}{\partial y^2} \quad (8)$$

Boussinesq approximation:

$$\rho = \rho_\infty [1 - \beta_T (T - T_\infty)] \quad (9)$$

Boundary conditions:

$$y = 0 : v = V_w, T = T_w \quad (10.1-3)$$

$$y \rightarrow \infty : u = 0, T = T_\infty \quad (11.1-3)$$

Here, u and v are the Darcian velocities in the x - and y -directions; μ , p and ρ are the variable viscosity, the pressure and the density of the fluid, respectively; K is the permeability of the porous medium; g is the gravitational acceleration; T is the volume-averaged temperature; α is the equivalent thermal diffusivity; β_T is the thermal expansion coefficients of the fluid; V_w is the uniform blowing/suction velocity.

The stream function ψ is defined by

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} \quad (12)$$

Thus, the continuity equation is automatically satisfied.

We note the governing equations (6) and (7). If we do the cross-differentiation $\partial(\mu \cdot u)/\partial y - \partial(\mu \cdot v)/\partial x$, then the pressure terms in equations (6) and (7) can be removed. Next step, with the help of the equation (9) and the boundary layer approximation ($\partial/\partial x \ll \partial/\partial y$), then we can obtain

$$\frac{\partial}{\partial y} (\mu \cdot u) = \rho_\infty g K \left[\beta_T \frac{\partial T}{\partial y} \right] \quad (13)$$

Integrating equation (13) once and via equations (1) and (11), then we get

$$u = [1 + \gamma(T - T_\infty)] \frac{\rho_\infty g K}{\mu_\infty} [\beta_T (T - T_\infty)] \quad (14)$$

Using the following dimensionless non-similarity variables:

$$\xi = \frac{2V_w \cdot x}{\alpha \cdot Ra_x^{1/2}} \quad (15.1)$$

$$\eta = \frac{y}{x} Ra_x^{1/2} \quad (15.2)$$

$$f(\xi, \eta) = \frac{\psi}{\alpha \cdot Ra_x^{1/2}} \quad (15.3)$$

$$\theta(\xi, \eta) = \frac{T - T_\infty}{T_w - T_\infty} \quad (15.4)$$

$$Ra_x = \frac{\rho_\infty g \beta_T K (T_w - T_\infty) \cdot x}{\mu_\infty \alpha} \quad (15.5)$$

Substituting equation (15) into equations (14), (8) to (11), we obtain

$$f' = (1 - \frac{\theta}{\theta_r}) \cdot \theta \quad (16)$$

$$\theta'' + \frac{1}{2} f \theta' = \frac{\xi}{2} (f' \cdot \frac{\partial \theta}{\partial \xi} - \theta' \cdot \frac{\partial f}{\partial \xi}) \quad (17)$$

The boundary conditions are defined as follows:

$$\eta = 0 : f = -\frac{\xi}{2}, \theta = 1 \quad (18)$$

$$\eta \rightarrow \infty : \theta = 0 \quad (19)$$

Furthermore, in terms of the new variables, the Darcian velocities in x - and y - directions are, respectively, given by

$$u = \frac{\alpha Ra_x}{x} f' \quad (20)$$

$$v = -\frac{\alpha Ra_x^{1/2}}{x} \left[\frac{1}{2} \cdot f + \frac{1}{2} (\xi \cdot \frac{\partial f}{\partial \xi} - \eta \cdot f') \right] \quad (21)$$

Where primes denote differentiation with respect to η . ξ defined in equation (15.1) is the surface blowing/suction parameter; equation (18) can be obtained by integrating equation (21) versus ξ once and by setting $\eta = 0$ (at the surface, $y = 0$, then $\eta = 0$), and with the help of equation

(10). On the one hand, for the case of blowing, $V_w > 0$ and hence $\xi > 0$. On the other hand, for the case of suction, $V_w < 0$, and hence, $\xi < 0$.

Via the equations (15.4) and (2), the viscosity-variation parameter θ_r is a constant and is defined by

$$\theta_r = \frac{T_r - T_\infty}{T_w - T_\infty} = -\frac{1}{\gamma(T_w - T_\infty)} \quad (22)$$

For equation (1), it is value referring that for $\gamma \rightarrow 0$, i.e.

(constant viscosity) then $\theta_r \rightarrow \infty$. It is also important to

note that θ_r is negative for liquid and positive for gas.

The result of practical interest in many applications is the surface heat transfer rate. The surface heat transfer rate is expressed in terms of the local Nusselt number

$Nu_x / Ra_x^{1/2}$ defined as following:

$$\frac{Nu_x}{Ra_x^{1/2}} = -\theta'(\xi, 0) \quad (23)$$

For the case of $\theta_r \rightarrow \infty$, equations (16) to (19) are reduced to these of Yih⁽²⁾ where a non-similar solution was obtained previously.

The analysis integrates the system of equations (16) to (19) by the implicit finite difference approximation together with the modified Keller box method of Cebeci and Bradshaw⁽⁸⁾. First, the partial differential equation is converted to a system of three first-order equation. Then, these first-order equations are expressed in finite difference form and solved by the iterative scheme along with their boundary conditions. This method provides a better convergence rate and reduces the numerical calculation times.

The initial size of the calculated grid is $\Delta\eta_1 = 0.01$, the variable grid increment parameter is set to 1.01, the maximum value of η_∞ is 15 and $\Delta\xi = 0.1$ (uniform grid). When the error value of θ'_w becomes less than 10^{-5} , the iteration process is stopped and the final temperature distribution is given.

2.2 Taguchi method

Phase 2: Taguchi method is used to simulate three different values of two parameters, A: blowing/suction parameter $\xi(-2, 0, 2)$, B: viscosity-variation parameter $\theta_r(-2, \infty, 2)$.

If arranged in full factor method to set the parameter, it will take quite a large computation time. Therefore, this experiment selected Taguchi method for numerical optimization design by numerical simulation can reduce times. This study was chosen $L_9(3^4)$ orthogonal table.

The two factors (A, B, 2 columns) have three levels (1, 2, 3), the number of simulations is 9 times. The levels of these two factors appear the same and this is a balance. This method will be used to establish the factor-level and preliminary data combinations as shown in Table 1. Select the Taguchi $L_9(3^4)$ orthogonal table via Table 1 as shown in Table 2.

Table 1. Factor - Level table.

Level		1	2	3
Factor	A	-2	0	2
	B	-2	∞	2

Table 2. Orthogonal table.

Row No.	1	2	3	4
1	1	1	1	1
2	1	2	2	2
3	1	3	3	3
4	2	1	2	3
5	2	2	3	1
6	2	3	1	2
7	3	1	3	2
8	3	2	1	3
9	3	3	2	1

The measure used in science and engineering is the signal to noise ratios. For this purpose, the signal to noise ratio (S/N ratio) measured with dB is used. Minimizing quality characteristics is equivalent to maximizing the S/N ratio, defined by

The Signal / Noise ratio:

$$S / N = -10 \cdot \log_{10} \frac{1}{n} \sum_{i=1}^n \left(\frac{1}{Y_i^2} \right) \quad (24)$$

Where Y_i denote local Nusselt number in Table 6.

3. Results and Discussion

In order to verify the accuracy of our current method, we compared our results with those of Minkowycz and Cheng⁽¹⁾ and Yih⁽²⁾. The results of comparing these articles are very consistent, as shown in Table 3.

Table 3. Comparison of the values of $-\theta'(\xi, 0)$ for various values of ξ with $\theta_r = \infty$.

ξ	Minkowycz and Cheng ⁽¹⁾	Yih ⁽²⁾	Present results
-2	1.0680	1.0725	1.0726
-1	—	—	0.7117
0	0.4438	0.4437	0.4437
1	—	—	0.2595
2	0.1423	0.1416	0.1411

The numerical results are presented for the blowing/suction parameter ξ ranging from -2 to 2 and the viscosity-variation parameter θ_r ranging from -2 to 2.

Figure 2 plots the effects of blowing/suction parameter ξ and the viscosity-variation parameter θ_r on the dimensionless temperature profile. For a fixed ξ , it is observed that the dimensionless temperature profile increases with increasing the viscosity-variation parameter θ_r from -2 (liquid) to 2 (gas), thus decreasing the dimensionless wall temperature gradient. For θ_r is given, as the blowing/suction parameter ξ is increased from -2 (suction) to 2 (blowing), the dimensionless wall temperature gradient has the tendency to reduce.

Fig 2. The dimensionless temperature profile for two values of ξ and θ_r .

The orthogonal table numerical of the local Nusselt number, the local Nusselt number of signal/noise ratio is shown in Table 4. Orthogonal table analysis by Taguchi analysis results (Tables 5). From the local Nusselt number and signal noise ratio, data can be obtained from the A factor is the level 1, B factor is the level 1.

It is confirmed that the value generated by the Taguchi method is the same as $\xi(-2)$ and $\theta_r(-2)$ in the orthogonal table 4. The maximum value of the local Nusselt number is 1.1152 for the $\xi(-2)$, $\theta_r(-2)$.

Table 6 lists the values of local Nusselt number for various values of ξ , θ_r . When ξ increases, the $-\theta'(\xi, 0)$ decreases. As the $\xi = -2$, the maximum value of the $-\theta'(\xi, 0)$ is 1.1152. It has been found that both the local Nusselt number decreases with increasing the viscosity-variation parameter θ_r . This is because for the case of $\theta_r > 0$ (gas) decreases the dimensionless surface temperature gradient, as shown in figure 2.

Table 4. The orthogonal table numerical of the local Nusselt number, the local Nusselt number of signal/noise ratio.

	local Nusselt number	Nusselt number signal noise ratio
$\xi(-2), \theta_r(-2)$	1.1152	0.9471
$\xi(-2), \theta_r(\infty)$	1.0726	0.6088
$\xi(-2), \theta_r(2)$	1.0342	0.2921
$\xi(0), \theta_r(-2)$	0.5180	-5.7134
$\xi(0), \theta_r(\infty)$	0.4437	-7.0582
$\xi(0), \theta_r(2)$	0.3536	-9.0298
$\xi(2), \theta_r(-2)$	0.2033	-13.8373
$\xi(2), \theta_r(\infty)$	0.1411	-17.0095
$\xi(2), \theta_r(2)$	0.0706	-23.0239
average value	0.5503	-8.2027

Table 5. The results of the local Nusselt number and signal noise ratio orthogonal table simulation.

Factor Level	Nusselt number signal noise ratio	
	A	B
1	0.6160	-6.2012
2	-7.2671	-7.8196
3	-17.9569	-10.5872
Delta	18.5729	4.3860
Ranking	1	1
Level	Mean Value	
1	1.0740	0.6122

2	0.4384	0.5525
3	0.1383	0.4861
Delta	0.9357	0.1261

Table 6. Values of $-\theta'(\xi, 0)$ for ξ , θ_r .

ξ	θ_r	$-\theta'(\xi, 0)$
-2	-2	1.1152
0	-2	0.5180
2	-2	0.2033
-2	-2	1.1152
-2	∞	1.0726
-2	2	1.0342
0	-2	0.5180
0	∞	0.4437
0	2	0.3536
2	-2	0.2033
2	∞	0.1411
2	2	0.0706

4. Conclusions

The conclusion of the first phase is as follows:

1. As the blowing/suction parameter ξ increases from -2 (suction) to 2 (blowing), the local Nusselt number decreases.

2. The viscosity-variation parameter θ_r enhances from -2 (liquid) to 2 (gas), the local Nusselt number reduces.

The second phase is to invoke the Taguchi experimental method to replace the traditional single factor experimental method. All parameter optimization design is to find the maximum local Nusselt number. The larger the local Nusselt number, the more amount of heat was taken away from the plate. The number of experiments can be greatly reduced, effectively saving the computer simulation time. Numerical computer simulation shows that the maximum values of local Nusselt number is 1.1152 for the case of $\xi(-2)$, $\theta_r(-2)$.

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