Design, Analysis, and Control of a Stiffness-Tunable 3-DOF Rubber-Bearing Positioning Stage

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Abstract

A 3-DOF θ_x - Y - θ_z rubber bearing stage with stiffness-adjustment capability is design, realized, and controlled in this work. The stiffness of rubbers can be adjusted easily by adding a preload from different directions. Base on elastomeric materials analysis, the stiffness of rubber and therefore dynamic characteristics of the stage can be changed with such a preload. In comparison with positioning stages designed by compliance mechanism approach, such characteristics could be an opportunity to improve the performance by replacing compliant bearing with rubber-bearing one. The dynamic testing results show that with a compression of 0.8mm on a 1.5mm thick rubber, the stiffness and natural frequency increased by 100% and 30% in Y -axis, 15% and 7% in θ_x -axis, and even by 510% and 140% in θ_z -axis, respectively. Associated PID and integral sliding mode controllers are then implemented for characterizing the position control performance. Currently, the rubber bearing stage can achieve a 79 Hz and 110 Hz controlling bandwidth by using PID and integral sliding mode controller (ISMC), respectively in Y -axis motion. In the future, the effect of different pre-compression displacement on dynamics characteristics of the stage, such as stiffness, natural frequency, damping ratio, will be tested. Additional control design in multi-axis coupling will be addressed to evaluate the performance in precision motion related applications.

Keywords: elastomeric bearing, positioning stage, feedback control.

1. Introduction

Positioning stage have been widely used in manufacturing system and precision metrology⁽¹⁾. For example, fast steering mirrors, which is used in laser manufacturing related applications, comprise rotational positioning stages⁽²⁾. Such stages required higher precision, high bandwidth, and high resolution for the purpose of enhancing manufacturing capability. Traditionally, these stages are usually designed and realized based on compliant mechanisms⁽³⁾. It provides stiffness by its compliant structure. For example, compliant stages usually utilize beam flexural deformation to gain the required compliance, which depends on the geometry and material properties. For example, for achieving a high compliant structure, the structural length could be long and is not adjustable once the structure is realized. If the dynamics characteristics of stage need to be adjusted, the shape of compliant structure would need to be re-design and this causes inconveniences.

Elastomeric bearings (aka rubber bearings) are essentially rubber-metallic laminates structures. They are used in building technologies to reduce vibration for many years⁽⁴⁾. Recently, this kind of elastomeric materials have been integrated into precision stage design. By changing the shape factor of rubbers, the stiffness under compression can be much higher than that in shearing direction⁽⁵⁾. With proper design, the strong stiffness anisotropy can be exhibited that the motion in shear direction can be flexible while the motion in compression direction be blocked effectively. However, elastomeric bearings are viscoelastic

material, its behavior depends on mechanical properties of materials, operating condition and geometry. The characteristics also influenced by temperature. It is important to perform essential characterization for fully taking the advantage of this approach.

For example, Kulk⁽⁶⁾ designed a fast steering mirror for optical communication. With elastomeric bearings, the system exhibited a stroke of 3.5-mrad angular motion with 10-kHz bandwidth. Teng⁽⁷⁾ developed a one-dimensional elastomeric bearing positioning stage that provide a 139μm stroke. With PID controller, the stage can have a 27-Hz bandwidth, the bandwidth is increased to 350 by using integral sliding mode controller (ISMC). In our previous work, a 3-DOF elastomeric bearing stage has also been developed⁽⁸⁾, the bandwidths are 80Hz, 62Hz and 54Hz in Y -, θ_x -and- θ_z axis. In that work, it also showed that the shearing stiffness and natural frequency could be adjustable with adding preload in compression direction⁽⁸⁾. This could be an advantage for rubber bearing stage that the dynamic model can be tuned easily. Therefore, it is important to design a stage with adjustable preloading for evaluating the effect of preloading on the characteristic dynamic and control performance systematically for future related applications.

The motivation of the research is, therefore, to realize a novel 3-DOF stage which can provide pre-load on rubber bearing in order to make its dynamics characteristics adjustable easily. In this work, we apply viscoelastic dynamics to developed system models of stage. Meanwhile, two controllers have also been designed to realized positioning experiment. Notice that although our previous investigation⁽⁸⁾ dealt with all possible deformation types, this work only focuses on shearing stiffness only and it leaves as our future work for exploring the rubber dynamics subjected to other types of pre-loads.

The research flow is shown in Fig. 1. First is to establish the rubber bearing stage with preload-adjustment, followed by dynamic characterization for obtaining the system model. Furthermore, the dynamic experiment with adding preload is demoed to show how the stage model dynamics be adjustable in Y -, θ_x -and- θ_z axis. Next, PID controller and integration sliding model controller (ISMC) has been designed and used for positioning experiment in Y -axis.

The rest of this article presents the work in detail. The conceptual design of the stage and realization are presented in Section 2. Next, system dynamic test is presented in Section 3. In Section 4, the design of the controller is

presented and followed by the positioning experiment addressed in Section 5. Finally, Section 6 concludes the paper.

Fig. 1. Research flow of stage realization.

2. Stage Design and Realization

2.1 3-DOF Positioning Stage

A 3-DOF rubber bearing positioning stage has been developed⁽⁸⁾. This stage consists of one translation (Y -axis) and two rotational (θ_x and θ_z -axis) DOF, the Schematic plot for stage design is shown in Fig. 2. As shown in Fig. 3a, three AVM40-20 voice coil motors (VCM) are used to actuate the stage. Three ASP-50-CTA capacitive probes are the sensor to capture the stage motion. Four rubber pads ($15 \times 15 \times 1.5 \text{ mm}^3$) are attached on an aluminum block ($30 \times 30 \times 30 \text{ mm}^3$) as the main body of the stage. The stiffness of the rubber bearing stage (k_y , k_{θ_x} and k_{θ_z}) is express by shearing stiffness(k_s), torsional stiffness(k_t) and bending stiffness(k_b)⁽⁸⁾.The stiffness of rubber bearing k_s , k_t and k_b are 120N/mm, 3.81Nm/rad and 11.25Nm/rad by testing method. The equation of motion of stage can be obtained as

$$M_y \ddot{Y} + C_y \dot{Y} + k_y Y = F_y \quad (1a)$$

$$I_{xx} \ddot{\theta}_x + C_x \dot{\theta}_x + k_{\theta_x} \theta_x = M_x \quad (1b)$$

$$I_{zz} \ddot{\theta}_z + C_z \dot{\theta}_z + k_{\theta_z} \theta_z = M_z \quad (1c)$$

The coordinates of force and measurement displacement need to be transformed by 3 voice coil motors (F_1 , F_2 and F_3) and three capacitive sensors (Y_1 , Y_2 and Y_3). The relations show in Chen's thesis⁽⁸⁾, it related to the arm length of force (a_f) and measurement point (a_y). Rubber bearing of this positioning stage cannot be adding preload,

which means the dynamics characteristics is un-adjustable. In contrast, a rubber-adjustment design has installed to the stage, as shown in Fig.3, four mechanisms are used to compress rubbers. This could make the dynamic characteristics of stage change.

Fig. 2. Schematic plot for stage design.

(a) schematic plot of stage.

(b) stage realization.

Fig. 3. Rubber bearing stage.

2.2 Rubber Stiffness Adjustment Design

The rubber stiffness is characterized by a material testing system, called HSC⁽⁸⁾, which provides a compression loading in compress direction of rubber by

step motor. It can test shear stiffness under different preload. The experiment data shows that with different preload, the shear stiffness increases furthermore, the natural frequency and damping ratio of rubber bearing also change with adding preload. With this result, we can see that the rubber can adjust easily with different loading condition.

A new 3-DOF rubber bearing stage has been developed, as Fig. 3 shown. Smaller VCMs AVM30-15 have been chosen to make the mass of stage lighter. The parameters of a_x and a_y are 62.5mm and 50mm. Fig. 4 shows the design that rubbers can be compress and fixed by screws in four directions. Compare to HSC system, not only shear stiffness relates to system dynamics, the bending stiffness and torsional stiffness also need to be considered. The entire control flow is shown in Fig. 5.

(a) preload adjustment design.

(b) preload adjustment realization.

Fig. 4. Rubber stiffness adjustment design.

Fig. 5. Block diagram for controlling the system.

3. System Dynamic Test

3.1 System Dynamics Modeling

The system model is established by step and sinusoidal response. Since rubbers are viscoelastic materials, thus a generalized Maxwell model is used to perform a time- and frequency-varying spring $K(s)$, which combined with three linear springs (k_1 , k_2 and k_3) and two dampers (c_2 and c_3)⁽⁹⁾. The stage model is shown as Fig. 6. The system model could be expressed as

$$G_{ij} = \frac{1}{M_{ij}s^2 + C_{ij}s + K_{ij}(s)} \times G_v \quad (2)$$

Here the time dependent stiffness, K_{ij} , can represent the creeping effect of rubber bearing stiffness⁽⁷⁾. G_v denotes the first-order order system of voice coil motor, the cut-off frequency ω_v is $300 \times 2\pi$, and G_{ij} represent the transfer function between i -axis output and j -axis input with the units of $\mu\text{m}/\text{V}$ for translation axis and mrad/V for rotation axis, respectively. The models of system are established by curve fitting. For example, the parameters of G_{yy} is shown in Table 1. Comparison of the simulation and the experimental results are shown in Fig. 7. The creep effect of rubber makes the stiffness of stage decrease.

The coupled effect also established by fitting step response and sinusoidal response. The transfer function models of coupled axis are much more complicated. Fig. 8 and Fig. 9 shows the simulation and experimental result of G_{xy} and G_{zy} .

Fig. 6. Stage viscoelastic model.

Table 1. The parameters of G_{yy} .

	M	C	k_1
	1.17×10^{-5}	0.0025	20.41
k_2	k_3	c_2	c_3
2.45	1.84	1.2245	0.1224

(a) Y-axis step response.

(b) bode diagram of Y-axis.

Fig. 7. Dynamics model of Y-axis.

Fig. 8. Dynamics model of θ_x -axis.

Fig. 9. Dynamics model of θ_z -axis.

3.2 Dynamic experiment with preload

In this part we add preload to compress rubber bearing and show how the dynamic models are influenced. The rubbers have been adding preload and compressed 0.8mm in θ_x -direction as shown in Fig. 9. In this situation, compress k_s makes k_y change, $k_{\theta x}$ is affect by compress k_s and k_t , and $k_{\theta z}$ is influenced by k_b and k_s . By sinusoidal response test, transfer function of G_{yy} , G_{xx} and G_{zz} has change, the results are shown in Fig. 10. The stiffness and natural frequency increase with adding preload. Table 2 shows the comparison of dynamic characteristics. G_{zz} has significant change in both stiffness and natural frequency, in the other hand, dynamic characteristic of G_{xx} does not alter so much.

Fig. 9. Compress rubber bearing condition.

Table 2. Characteristics of rubber bearing stage.

	Stiffness($V_i/\mu\text{m}, V_i/\text{mrad}$) ($i=y,x,z$)		Natural frequency(Hz)	
	original	Compress 0.8mm	original	Compress 0.8mm
Y	0.183	0.367	225	295
θ_x	0.441	0.503	75	80
θ_z	0.733	4.481	80	190

(a) Y -axis response.

(b) θ_x -axis response.

(c) θ_z -axis response.

Fig. 10. Bode diagram with adding preload.

4. Controller Design

There are two controllers has used for stage control: PID controller and integral sliding mode control. PID control is selected to be a basic threshold of control performance. After PID controller established, integral sliding mode control will be applied to seek a better control result.

4.1 PID control

PID controller is widely used in control application⁽¹⁰⁾. With appropriate design, the resonance peak of bode diagram can be suppressed effectively. The traditional PID controller are written as

$$G_{pid} = k_p \left(+ \frac{1}{T_i s} + T_d s \right) \quad (3)$$

Where k_p is proportional gain, T_i is integral gain and T_d is differential gain.

4.2 Integral sliding mode control

On the other hand, the integral sliding mode control (ISMC) is also be designed⁽¹¹⁾. A sliding function $s(x,t)$ has been chosen, which is defined as

$$s(x,t) = \left(\frac{d}{dt} + \lambda \right)^{n-1} \tilde{x} + \beta \int_0^t \tilde{x} \quad (4)$$

Where $\tilde{x} = x(t) - x_d(t)$. x_d is the command of stage position. There are four parameters of ISMC: λ , η , ϕ and β . Please refer to Deng's work⁽⁷⁾ for the detail in determining of ISMC's control parameters.

5. Control Result

The NI cRIO-9014 FPGA is chosen as the real time controlling platform for developing controller to conduct positioning experiment. The sampling rate (F_s) of system is 10-kHz. A 15.7 μm step input in Y -axis direction is command. Step response and sinusoidal response are tested.

5.1 Comparison with different dynamic model

There are 3 control result with PID controller shown in Fig. 11. First, we control the stage with PID gain $k_p=3$, $T_i=0.7/F_s$ and $T_d=0.5/F_s$ can reach a rise time and settling time of 6ms and 25.3ms with a 5.1% overshoot (PID-a).

Second, the rubbers were compressed 0.8mm in θ_x -direction without adjust PID gain (PID-b). The result has different due to the plant change. The rise time and settling time are 11.8ms and 41.4ms in this condition. Final, another PID gain ($k_p=4.5$, $T_i=0.73/F_s$ and $T_d=0.02/F_s$) has been chosen to make the performance better, it can achieve a rise time and settling time of 5.8ms and 24.3ms in current work. Fig. 12 shows the sinusoidal test result. The -3dB bandwidths are 79Hz, 35Hz and 79Hz in these 3 conditions. From PID-a and PID-b, the bode plot of close loop system are different because the dynamic model has changed. It shows that PID-c can reach a same bandwidth as that of PID-a, due to the increase of natural frequency from 210 Hz to 250 Hz with preload.

Fig. 11. Step responses (PID-a, -b and-c).

Fig. 12. Bode plot of the close-loop system (PID-a, -b and-c).

5.2 Comparison with different controller

On the other hand, original plant with ISMC has been selected to compare with PID-a. ISMC with a parameter set of $\lambda=0.0018$, $\eta=50$, $\phi=0.005$ and $\beta=8$ reaches a rise time of 3.9ms and settling time of 20.9ms, which is shown in Fig. 13. The sinusoidal test result is shown in Fig. 14, the -3dB bandwidth is 110Hz by using ISMC. Table 3 summarizes the performance of the control result in these setups. In current works, the ISMC has better settling time and bandwidth in Y -axis. The θ_x and θ_z axis has steady state error because only one controller is applied in command axis(Y -axis). In future work, controllers of θ_x and θ_z axis will be designed and added in control system to decrease steady state errors which cause by coupled effect.

Fig. 13. Step responses (PID-a and ISMC).

Fig. 14. Bode plot of the close-loop system (PID-a and ISMC).

Table 3. Performance of the system under control.

	PID-a	PID-b	PID-c	ISMC
Rise time (ms)	6	11.8	5.8	3.9
Settling time (ms)	25.3	41.4	24.3	20.9
Overshoot (%)	5.1	4.4	7.4	3.3
Bandwidth (Hz)	79	35	79	110

6. Conclusions

In this work, a stiffness-tunable 3-DOF rubber bearing positioning stage has been developed, the design and control of the stage is presented for dynamic characteristic adjustment and positioning control. The system is actuated by 3 voice coil motor. 3 capacitive probes are used to sense one translational and two rotational DOF. PID controller and ISMC are designed for performing the motion control. Currently, the stiffness-tunable design shows that the dynamic characteristics of this stage is adjustable. This is an advantage compare with compliant stage. For example, the length of compliant mechanism needs to be decreased by 30% to make the stiffness increase by 100%, that makes the size of compliant stage have large adjustment. The stiffness of the rubber bearing stage can increase by 100% and by 510% in translational direction and in rotational direction with small scale adjustment, which is more efficiently than compliant stage. The -3dB bandwidth of controlling Y -axis is 79Hz by PID control. With ISMC, the bandwidth can reach 110Hz. In current work the stage with rubber adjustment can approaching same control result to original stage. The future work of this research is to finish controller design of all three axes to realize coupling control. θ_x and θ_z -axis control experiment with preload will get on. It could improve control performance with θ_x and θ_z -axis control testing. With further future, this rubber bearing stage should be useful for various applications such as precision engineering or laser scanning.

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