Experimental Stress Intensity Factor Analysis of Mode II using Strain Gauge and Tensile Shear Plate Specimen

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Abstract

In this study, analysis of the stress intensity factor ($K_{II}$) in the in-plane shear mode was examined using a 2-axis orthogonal strain gauge. A specially shaped tensile shear plate specimen under shear control around the crack was prepared. The shearing strain near the crack was measured with this specimen and strain gauge, and the $K_{II}$ was calculated from the analytical equation. The $K_{II}$ by using the FEM analysis result and the extrapolation method was regarded as the theoretical value and it was compared with the experimental value. In addition, the effects of crack length and distance between both cracks on the analysis accuracy of the $K_{II}$ were investigated. As a result, it was confirmed that the $K_{II}$ analysis in this study showed good accuracy except for some parts, suggesting the possibility of the proposed method.

Keywords: Stress Intensity Factor, In-plane Shear Crack, Strain Gauge, Tensile Shear Plate Specimen, FEM

1. Introduction

In recent years, many serious incidents caused by cracks in structures have been reported. In particular, the problem of aging social infrastructure such as tunnels and bridges has become apparent. In order to solve these problems, it is important to evaluate the risk of cracking and take appropriate measures to ensure safety. As a method for evaluating the risk of cracks, it is common to introduce the stress intensity factor ($K$), which is a fracture mechanics parameter. Therefore, researches using strain gauges specialized for $K$ value analysis and studies on $K$ value analysis which use commercially available strain gauges have been conducted for some time. In such previous studies, Kurosaki et al. analyzed the $K$ value for Mode II (that is, $K_{II}$) using a 2-axis orthogonal strain gauge, and obtained certain results. However, the disadvantage of strain gauges is that they require a lot of work and time to use, and a simpler and labor-saving measurement method is desired in the field. In this regard, we have been studying a method for analyzing the $K_{II}$ value of the in-plane shear mode by using a strain checker that can save labor in strain measurement and a tensile shear plate specimen. The strain checker (Tokyo Measuring Instruments Lab., Co., Ltd., FGMH-3A) has a built-in frictional strain gauge. Since it is attached to the measurement target by the magnetic force of the main body, it can be used repeatedly. In the previous studies, the crack distance (shear length) of the tensile shear plate specimen was treated as constant, and the influence of the magnitude of the shear load around the crack has not been investigated.

Following the previous studies, we attempted to analyze the $K_{II}$ value of the in-plane shear mode using a 2-axis orthogonal strain gauge and five types of the tensile shear plate specimens with varying crack lengths and distances between cracks. The theoretical $K_{II}$ values were obtained using the FEM analysis results and the
extrapolation method, and then it was compared with the $K_{II}$ value obtained from the experimental results. From the results above, the effects of crack lengths and distances between cracks on the analysis accuracy of $K_{II}$ values were clarified.

2. Strain Component for In-plane Shear Mode at Crack Tip

Figure 1 shows the case where the crack tip is in the in-plane shear load mode. J. W. Dally et al.\(^{(13)}\) represent the stress component and strain component at point P, dividing them into the regions shown in Figure 1 for the crack tip region under such a load condition. In other words, the stress and strain equations are expressed with the first region governed by the singular term of the stress intensity factor, the outer region as the second region, and the outer region as the third region. In this study, which targets strain gauges with a finite length, the strain components up to the first and second regions are targeted. Moreover, if the number of expansion terms of the strain component equation are taken large and expressed up to the third term, it becomes the following equation.

\[
E \varepsilon_{x-shear} = -C_0 r^{-1/2} \sin \frac{\theta}{2} \left[ 2 + (1 + \nu) \cos \frac{\theta}{2} \cos \frac{3 \theta}{2} \right] \\
+ C_1 r^{1/2} \sin \frac{\theta}{2} \left[ 2 + (1 + \nu) \cos^2 \frac{\theta}{2} \right] \cdots (1)
\]

\[
E \varepsilon_{y-shear} = C_0 r^{-1/2} \sin \frac{\theta}{2} \left[ 2 \nu + (1 + \nu) \cos \frac{\theta}{2} \cos \frac{3 \theta}{2} \right] \\
- C_1 r^{1/2} \sin \frac{\theta}{2} \left[ 2 \nu + (1 + \nu) \cos^2 \frac{\theta}{2} \right] \cdots (2)
\]

\[
G \gamma_{xy-shear} = C_0 r^{-1/2} \cos \frac{\theta}{2} \left[ 1 - \sin \frac{\theta}{2} \sin \frac{3 \theta}{2} \right] \\
+ C_1 r^{1/2} \cos \frac{\theta}{2} \left( \sin^2 \frac{\theta}{2} + 1 \right) \cdots (3)
\]

Since $r^{0/2}$ in the second term on the right-hand side of the above equation is zero, it does not appear in each of $E \varepsilon_{x-shear}$, $E \varepsilon_{y-shear}$ and $G \gamma_{xy-shear}$. Therefore, in equations (1)-(3), the third term of the right-hand side $r^{1/2}$ appears after the first term. $C_0$ and $C_1$ in these equations are unknown coefficients. In each equation (1)-(3), the coefficient $C_0$ of the first term on the right-hand side is expressed by the following equation (4) including the shear mode $K_{II}$.

\[
C_0 = \frac{K_{II}}{\sqrt{2\pi r}} \cdots (4)
\]

The following are two equations in which $\theta = 0$ is substituted into equation (3) and the shear strain component $G \gamma_{xy-shear}$ is adopted up to the second and third terms.

1. A case where up to the second term is adopted (Since $r^{0/2}$ in the second term is zero, it is represented only by the first term).

\[
G \gamma_{xy-shear} = \frac{K_{II}}{\sqrt{2\pi r}} \cdots (5)
\]

II. When up to the third term is adopted (Since the second term is zero from the above, the number of terms is represented by two).

\[
G \gamma_{xy-shear} = \frac{K_{II}}{\sqrt{2\pi r}} + C_1 \sqrt{r} \cdots (6)
\]

3. Analysis of Stress Intensity Factor $K_{II}$ for In-plane Shear Mode

3.1 Case of Analysis using Strain Gauge

Solving the above equation (5) for $K_{II}$ gives the following equation (7).

\[
K_{II} = G \gamma_{xy-shear} \sqrt{2\pi r} \cdots (7)
\]

Equation (7) was used as an analytical equation by Kurosaki et al.\(^{(7-9)}\) and Shimura et al.\(^{(11, 12)}\) in previous studies, and this...
study also deals with it. As shown in Figure 2, a 2-axis orthogonal strain gauge (Kyowa Electric Co., Ltd., KFGS-1-120-D16-11, Gauge length is 1 mm) is bonded to the extension line of the crack at an angle of ±45°. Then, the distance \( r \) from the crack tip becomes 1.3 mm. With this strain gauge, normal strain \( \varepsilon \) is measured twice (\( = 2\varepsilon \)), and that value can be directly treated as a shear strain \( \gamma_{xy\text{-shear}} \). By substituting the value of the shearing strain \( \gamma_{xy\text{-shear}} \) and the distance \( r = 1.3 \) mm from the crack tip into equation (7), \( K_{II} \) can be calculated.

### 3.2 Case of Analysis by Finite Element Method

Figure 3 shows the dimensions and shape of the tensile shear plate specimen used in the accuracy verification experiment described later. Finite element analysis (FEM) is performed assuming the experimental situation of this specimen, and \( K_{II} \) is extrapolated using the element shearing strain of the analytical result by FEM near the crack tip. In this case, applying equation (6) considering up to the third term of the shearing strain component equation, and multiplying both sides of equation (6) by \( \sqrt{2\pi r} \), the analysis equation becomes as follows.

\[
G \gamma_{xy\text{-shear}} \sqrt{2\pi r} = K_{II} + \sqrt{2\pi C_1 r} \quad \cdots (8)
\]

When the left side of equation (8) is set to \( F \) and the coefficients other than \( r \) in the second term of the right side are rewritten as \( C \left( = \sqrt{2\pi C_1} \right) \), the equation is as follows.

\[
F = K_{II} + Cr \quad \cdots (9)
\]

The above-mentioned equation (9) is a linear function for \( r \), and the intercept is the stress intensity factor. That is, as shown in Figure 4, \( K_{II} \) can be obtained by extrapolating the \( F \) value on the left side with \( r \).

As the boundary conditions of the FEM model, constraints other than the \( x, y, z \)-axis translational displacement for the left pin-joining circular hole and the \( x \)-axis translational displacement for the right circular hole are applied as shown in Figure 3. In addition, a load in the \( x \)-axis direction (maximum 2000N) was provided to the circular hole for pin-joining on the right side. The material property values are defined as 205 GPa for Young's modulus \( E \) and 0.3 for Poisson's ratio \( \nu \), assuming the use of cold-rolled steel SPCC. In the element division, the basic shape is a tetrahedral primary element, and a hexahedral primary element is adopted in the parts where the element strains around the crack end are acquired. A minimum element size of about 0.05 mm was used for the region where stress concentration is expected.

### 4. Accuracy Verification Experiment

Figure 3 shows the specimen used in this study. Previous study by Miyagawa et al.\(^{(10)}\) have confirmed that the shape of this specimen is dominated by shearing stress between cracks in the central part. The specimen material is cold-rolled steel SPCC, and the red line in Figure 3 means the slits with a width of 0.32 mm due to wire cutting. These tips were regarded as the artificial cracks. The strain gauge was bonded to the crack tip as shown in Figure 2. Since it is a specimen shape with two cracks, it can be bonded to the opposite side. Furthermore, by using the back surface, it is possible to apply strain gauges at four locations with one specimen. This specimen was attached to a universal material testing machine (A&D Co. Ltd., TENSILON RTG-1310), and a tensile load was applied by displacement control of 0.001 mm/sec. The strains near the crack tips under the load were detected every 500N with the 2-axis orthogonal strain gauges and recorded on a data logger (Tokyo Measuring Instruments Lab., Co., Ltd., TDS-540). The strain measurement was performed three times for each specimen, and the arithmetic mean value of those strains was applied to the analytical equation (7) to calculate the \( K_{II} \).

<table>
<thead>
<tr>
<th>Specimen No.</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance between cracks ( L ) [mm]</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>15</td>
<td>10</td>
</tr>
<tr>
<td>Crack length ( a ) [mm]</td>
<td>10</td>
<td>15</td>
<td>20</td>
<td>15</td>
<td>15</td>
</tr>
</tbody>
</table>
value. In order to investigate the effects of crack length $a$ and crack distance $L$ on the analysis accuracy of $K_{II}$ values, the five types of specimens shown in Table 1 were used.

5. Results and Discussion

Figure 5 shows the results of strain measurement when the tensile loads were applied to the specimens (No.2, 4, 5) in increments of 500 N up to 2000 N. These results were obtained by fixing the crack length $a$ at 15 mm and changing the distance between both cracks $L$ to 10, 15, 20 mm. The abscissa is the loaded shearing stress $\tau$ (load $P$/shearing area $A$, MPa). In either case, it can be seen that the output of the shearing strain $\gamma_{xy}\text{shear}$ with respect to the loaded shearing stress $\tau$ has linearity. It can also be seen that the plot points of each symbol mean the loading level, and the shearing strain $\gamma_{xy}\text{shear}$ increases as the ratio $a/L$ of the crack length $a$ to the distance between both cracks $L$ increases. This is because when the shearing area $A$ (distance $L$ × specimen thickness $t$, mm$^2$) decreases, a large shearing stress $\tau$ occurs. In addition, when compared under the same loaded shearing stress $\tau$, it can be confirmed that $\gamma_{xy}\text{shear}$ increases as the ratio $a/L$ decreases.

Figure 6 shows $K_{II}\text{exp.}$, calculated using the shearing strain $\gamma_{xy}\text{shear}$ shown in Fig. 5 measured in the accuracy verification experiment and Eq. (7). In Fig. 6, $K_{II}\text{exp.}$ is taken on the abscissa. As the ratio $a/L$ increases, the distance between both cracks $L$ decreases, so the applied shear stress $\tau$ becomes the highest when $a/L = 1.5$. Therefore, it can be seen that $K_{II}\text{exp.}$ shows the maximum value when the ratio $a/L = 1.5$ among the three types in Fig. 6. Furthermore, as described above, the shearing strain $\gamma_{xy}\text{shear}$ increases as $a/L$ decreases under the same loaded shearing stress $\tau$, so it can be recognized that the $K_{II}\text{exp.}$ value increases due to the nature of analytical Eq. (7).

Figure 7 is a diagram for extrapolating the $K_{II}$ value from the $F$ value calculated by Eq. (9) using the element shearing strain $\gamma_{xy}$ obtained from the FEM result assuming the specimen under tensile load. As in the experiment, the assumed shearing load has four stages, and four types are plotted in this figure. The intersection (intercept) with the abscissa $F$ is the extrapolated value of the $K_{II}$ value, and these values were regarded as theoretical values $K_{II}\text{the.}$ in this study. The $K_{II}\text{the.}$ values for each load level are 1.558, 3.116, 4.675, 6.223 MPa$\sqrt{m}$, respectively. It should be noted that these values are for $a/L = 0.75$ (specimen No. 2, $a = 15$, $L = 20$). Table 2 shows the $K_{II}\text{the.}$ of all the remaining specimens obtained and organized by the same process. In this table, the loads applied to each specimen, the corresponding shearing stresses $\tau$ (upper column), and the theoretical values $K_{II}\text{the.}$ (lower column, bold) are also shown.

Using the above results, the error rate $e$ was calculated from the following equation to verify the analysis accuracy of the $K_{II}\text{exp.}$ obtained by the proposed method of this study.

\[
e = \frac{|K_{II}\text{exp.} - K_{II}\text{the.}|}{K_{II}\text{the.}}
\]
\[ e = \frac{K_{II \text{exp}} - K_{II \text{the.}}}{K_{II \text{the.}}} \times 100 \% \] \quad \ldots (10)

Figure 8 shows the relationship between the applied shearing stress \( \tau \) and the error rate \( e \) for each specimen. Except for \( a / L = 0.5 \) (specimen No. 1, \( a = 10, L = 20 \)), the error rate \( e \) is within \( \pm 10 \% \), indicating that it has excellent analysis accuracy. According to Miyagawa et al.\(^{(10)}\), when the ratio of the half width \( w \) of the specimen to the shear length (distance between both cracks \( L \)) is 3 or more, shear failure becomes dominant. In the geometry used this time, the only specimen that matches the condition is No. 5 (\( a / L = 1.5, a = 15, L = 10 \)) and \( w / L = 3.0 \) (\( w = 30, L = 10 \)). In the case of targeting within the elastic region as in this study, it is considered that the condition does not necessarily have to be satisfied. The reason why the error rate becomes large when \( a / L = 0.5 \) (specimen No. 1, \( a = 10, L = 20 \)) is not clear at present. However, it is inferred that the magnitude relationship between the crack length \( a \) and the distance \( L \) has an effect on the \( K_{II \text{the.}} \) analysis accuracy.

Therefore, the relationship between the ratio \( a / L \) to the crack length \( a \) and the distance between both cracks \( L \) and the error rate \( e \) is summarized in Fig. 9. In Fig. 9, the ratio \( a / L \) is taken on the abscissa, and the error rates \( e \) for the four ratios \( a / L \) (0.5, 0.75, 1.0 and 1.5) are plotted. Furthermore, when \( a / L = 1.0 \), the acting shear stress \( \tau \) is 7 types (11, 14, 22, 29, 33, 43, 58 MPa), so the number of plots shows the maximum of 7 points. When \( a / L = 0.5 \), the error rate exceeds 20% at any loaded shearing stress level, but in other cases, it can be confirmed that the error rate is within \( \pm 10 \% \).

### Table 2. Theoretical \( K_{II \text{the.}} \) for each specimen

<table>
<thead>
<tr>
<th>Specimen No.</th>
<th>Load ( F ) [N]</th>
<th>Applied shearing stress ( \tau ) [MPa]</th>
<th>Stress intensity factor ( K_{II \text{the.}} ) [MPa√m]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>500</td>
<td>1000</td>
<td>1500</td>
</tr>
<tr>
<td>(( a / L ))</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>11</td>
<td>22</td>
<td>33</td>
</tr>
<tr>
<td>(0.5)</td>
<td>1.560</td>
<td>3.119</td>
<td>4.677</td>
</tr>
<tr>
<td>2</td>
<td>11</td>
<td>22</td>
<td>33</td>
</tr>
<tr>
<td>(0.75)</td>
<td>1.558</td>
<td>3.116</td>
<td>4.675</td>
</tr>
<tr>
<td>3</td>
<td>11</td>
<td>22</td>
<td>33</td>
</tr>
<tr>
<td>(1.0)</td>
<td>1.561</td>
<td>3.121</td>
<td>4.679</td>
</tr>
<tr>
<td>4</td>
<td>14</td>
<td>29</td>
<td>43</td>
</tr>
<tr>
<td>(1.0)</td>
<td>1.702</td>
<td>3.405</td>
<td>5.107</td>
</tr>
<tr>
<td>5</td>
<td>22</td>
<td>43</td>
<td>65</td>
</tr>
<tr>
<td>(1.5)</td>
<td>1.995</td>
<td>3.910</td>
<td>5.866</td>
</tr>
</tbody>
</table>

Fig. 8. Verification result of accuracy for \( K_{II \text{the.}} \) analysis

As a result, the \( K_{II \text{the.}} \) analysis method proposed in this study suggests the range of \( a / L = 0.75 \) to 1.5 is currently useful.

### 6. Conclusions

In this study, we attempted to analyze the stress intensity factor \( K_{II \text{exp.}} \) in the in-plane shear mode using the 2-axis orthogonal strain gauges and the tensile shear plate specimens. Furthermore, the \( K_{II \text{exp.}} \) analysis accuracy was verified by comparing with the theoretical values \( K_{II \text{the.}} \) obtained by the extrapolation method using the results of FEM. In addition, the effects of crack lengths \( a \) and the distances between both cracks \( L \) on the analysis accuracy of \( K_{II \text{exp.}} \) were researched. The results and findings obtained so far are as follows.

1) \( K_{II \text{the.}} \) analysis method using the 2-axis orthogonal strain gauges and the tensile shear plate specimens was proposed.

2) Theoretical values \( K_{II \text{the.}} \) for the tensile shear plate specimens were obtained using FEM analysis and extrapolation method.
3) It was confirmed that the $K_{II \text{exp.}}$ analysis accuracy of the proposed method was within ± 10% except for some specimens. This suggests that the ratio $a/L$ to the crack length $a$ and the distance between cracks $L$ is useful in the range of 0.75 to 1.5.

Acknowledgment

This work was supported by JSPS KAKENHI Grant Number JP20K05009. We also gratefully acknowledge the work of past members of our laboratory and Manufacturing center first group belonging to National institute of technology, Tokyo campus.

References