Analysis on the Reconstruction of Computer-generated Hologram According to its Square Structure

Yuki Misaki^{a,*}, Yosuke Koga^a, Shiyuan Yang^a, Seiichi Serikawa^a

^aKyusyu Institute of Technology, 1-1 Sensui-cho Tobata-ku Kitakyuusyuu-shi Fukuoka-ken, 804-8550, Japan

*Corresponding Author. n349432y@mail.kyutech.jp

Abstract

Creation of an actual hologram serves as the structure of giving fixed area to the calculated hologram distribution data. For actual optical reproduction, the sinc function corresponding to a reproduction image is generally applied. However, in computer-generated simulation, when calculating hologram data, the sinc function is not taken into consideration. In this paper, analyzed about the reproduction image distribution in actual optical reproduction. As a result, it has checked that a sinc function started a reproduction image approximately by increasing sampling mark.

Keywords: hologram, sinc function, computer-generated hologram.

1. Introduction

Usually, manufacture of a hologram serves as the structure of giving fixed area to the calculated hologram distribution data. Therefore, in optical reproduction, the sinc function corresponding to a reproduction image is applied. However, in the general simulation, when calculating hologram data, the sinc function is not taken into consideration. Then, this research analyzes about the reproduction image distribution in optical reproduction, and the algorithm of the reproduction image in a simulation is shown.

2. Principle of the hologram

Fig.1 shows Fourier transformation type hologram playback optical system. Two-dimensional intensity distribution should represent in $f(x, y)$, sampled data in $f(m, n)$, hologram distribution data calculate by using $f(m, n)$.

Hologram distribution data represented as $W(k, l)$. Amplitude distribution of reconstructed image in the sampled point is represented by the following equation.

$$g(m, n) = \frac{1}{N^2} \sum_{k=-\frac{N}{2}}^{\frac{N}{2}-1} \sum_{l=-\frac{N}{2}}^{\frac{N}{2}-1} W(k, l) \exp\left[-\frac{j2\pi}{N}(mk + nl)\right] \quad (1)$$

Where, N is the number of the sampled dispersed points.

Generally, when manufacturing hologram, it becomes constant area distribution using the hologram distribution data $W(k, l)$. If this is expressed with one dimension, it will become like Fig.2.

Fig.1. Fourier transformation type optical setup

Fig.2. Amplitude distribution of the hologram

Fig.3. Two-dimensional notation figure of hologram side

Change discrete value into continuous value in order to have constant area. Therefore, collapse a rectangle wave in a discrete value.

$$H(u) = W(k) * \text{rect}(u) \quad (2)$$

Where k is the hologram data number, $W(k)$ has a value of $W\left(\frac{u}{\Delta u}\right)$, when u is an integral multiple of Δu , and it becomes zero except it. Moreover, $\text{rect}(u)$ expresses a rectangular wave, the range of $(-\frac{\Delta u}{2} \leq u \leq \frac{\Delta u}{2})$ has a value, and other ranges are 0. The two-dimensional notation of $H(u)$ is shown in the following formula. Moreover, the two-dimensional notation figure of a hologram side is shown in Fig. 3.

$$H(u, v) = W(k, l) * \text{rect}(u, v) \quad (3)$$

A reproduction image is obtained by Fourier-transform as.

$$\begin{aligned} g'(x, y) &= \iint H(u, v) \exp\{-j(xu + yv)\} du dv \\ &= \iint W(k, l) \exp[-j(xu + yv)] du dv \\ &\quad \times \text{sinc}(x, y) \end{aligned} \quad (4)$$

Reproduction image is obtained by Fourier transform $H(u)$. This become multiply square wave Fourier transform and Fourier transformation of hologram size distribution data. Because the sinc function is the Fourier transform of the rectangular wave, would put the sinc function in the reconstructed image. Therefore, in optical reproduction, the reproduction image in the sampling point of Δx interval serves as the following formula.

$$\begin{aligned} g'(m, n) &= \left\{ \frac{1}{N^2} \sum_{k=-\frac{N}{2}}^{\frac{N}{2}-1} \sum_{l=-\frac{N}{2}}^{\frac{N}{2}-1} W(k, l) \exp\left[-\frac{j2\pi}{N}(mk + nl)\right] \right\} \\ &\quad \times \text{sinc}(m, n) \\ &= g(m, n) \times \text{sinc}(m, n) \end{aligned} \quad (5)$$

3. Dependence of the sampling mark of the hologram side in a computation simulation

The reproduction image of the usual simulation is denoted by the formula (1). In this case, one mass of Fig. 3 is treated as a discrete point of one point. However, the reproduction image in consideration of actual hologram serves as a formula (5). In this case, hologram distribution is treated as continuous distribution. Then, one mass of Fig. 3 is subdivided and the mark of the discrete point of hologram increased. The square distribution of one mass of Fig. 3 cannot be correctly expressed with the case of one usual point. The effect of the sinc function in the reproduction image of the continuous distribution of hologram can be given by increasing the sampling mark in one mass. The hologram side in 1×1 point is shown in Fig. 4, and the hologram side at the time of subdividing one mass of Fig. 4 and considering it as 2×2 points is shown in Fig. 5.

In the following, we introduce the expression in the case of a $KN \times KN$ point. The hologram distribution data in this case is denoted by the following formula.

$$W_K(K \times k + p, K \times l + q) = W(k, l) \quad (6)$$

However, p and q are the following ranges.

$$p = 0, 1, 2, \dots, K - 1 \quad (7)$$

$$q = 0, 1, 2, \dots, K - 1 \quad (8)$$

1	2	3
4	5	6
7	8	9

Fig.4. The hologram side in 1×1 point

1	1	2	2	3	3
1	1	2	2	3	3
4	4	5	5	6	6
4	4	5	5	6	6
7	7	8	8	9	9
7	7	8	8	9	9

Fig.5. The hologram side in 2×2 point

k' and l' are made into the following formula and a reproduction image is denoted by $g_k(m, n)$.

$$k' = K \times k + p \quad (9)$$

$$l' = K \times l + q \quad (10)$$

$$\begin{aligned} g_k(m, n) &= \frac{1}{K^2 N^2} \sum_{k'=-\frac{KN}{2}}^{\frac{KN}{2}-1} \sum_{l'=-\frac{KN}{2}}^{\frac{KN}{2}-1} [W_K(k', l') \\ &\quad \times \exp\{-j \frac{2\pi}{KN} (k'm + l'n)\}] \\ &= \frac{1}{K^2 N^2} \sum_{k=-\frac{N}{2}}^{\frac{N}{2}-1} \sum_{l=-\frac{N}{2}}^{\frac{N}{2}-1} \sum_{k'=-\frac{K}{2}}^{\frac{K}{2}-1} \sum_{l'=-\frac{K}{2}}^{\frac{K}{2}-1} [W_1(K \times k + k', K \times l + l') \\ &\quad \times \exp\{-j \frac{2\pi}{KN} (K \times k + k')m + (K \times l + l')n\}] \\ &= g(m, n) \times \frac{1}{K^2} \sum_{k'=-\frac{K}{2}}^{\frac{K}{2}-1} \sum_{l'=-\frac{K}{2}}^{\frac{K}{2}-1} \exp\{-j \frac{2\pi}{KN} (k'm + l'n)\} \end{aligned} \quad (11)$$

4. Simulation and results

In this section, we analyzed the performance of the proposed method. Next, the experimental details are shown as following.

- Images 256×256pixel
- Initial phase Zero phase

An original picture image uses the square image of Fig. 6. In order that this simulation may avoid the influence of other errors, quantization or band limiting of the hologram are not performed. The reproduction image of $K=1$, Fig.7 and the reproduction image of $K=4$ (One cycle), and the reproduction image of Fig.8 and $K=16$ (One cycle) point are shown in Fig.9, and the relation between the maximum error and mark is shown in Fig.10.

Fig.6. Original image

Fig.7. The reproduction image in $K=1$

Fig.9. The reproduction image in $K=4$ (One cycle)

Fig.10. The reproduction image in $K=16$ (One cycle)

Fig.10. Relation between the maximum error and the mark K

The maximum error in $K=1$ was -11.68 [dB]. The maximum error in $K=2$ was -24.73 [dB]. The maximum error in $K=4$ was -37.04 [dB]. The maximum error in $K=8$ was -49.16 [dB]. The maximum error in $K=16$ was -61.21 [dB]. In the case of $K=1$ of Fig.7, from the result of the reproduction image, the surroundings

are white compared with the central part of an image. This is in the state where a surrounding light is strong. That is, the error has occurred. That a sinc function starts a reproduction image approximately has checked by Fig.7, Fig.8, and Fig.9 by increasing sampling mark. Therefore, the effect of a sinc function is brought about by fully increasing sampling mark.

5. Conclusion

In this research, it analyzed about the reproduction image distribution in actual optical reproduction. We proposed and the algorithm of the reproduction image in the simulation was shown. Usually, although the reproduction image needed to be multiplied by the sinc function in the simulation, $K=1, 4$, and the reproduction image of 16 were made to indicate that a sinc function starts a reproduction image approximately by increasing sampling mark in a simulation, and the value of the maximum error was made into the graph and checked.

References

- (1) Toshinori Hora, and Shiyuan Yang : “Iterative dummy area method with flexible dummy area size for the design of kinoform” , Proc. of the 1st International Conference on Industrial Applications Engineering 2013, pp.269-272, Kitakyushu, Japan.
- (2) Toyohiko Yatagai: “hikaritohuuriehennkan(in Japanese)”, 1986
- (3) L.B.Lesem, P.M. Hirsch, J.A. Jordan: “The Kinoform: a new wavefront reconstruction device”, IBM Journal of Research and Development, Vol.13, No.2, pp.150-155, 1969.