Application of Wavelet Analysis in Random Drift Signal Filtering of Fiber Optic Gyroscope

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Abstract

In order to remove the noise from random drift signal of fiber optic gyroscope (FOG), the wavelet analysis technique is proposed in the filter Processing. Based on the component elements of random drift signal of FOG, mathematical model of random drift signal of FOG is built. According to the shortcoming of hard threshold and soft threshold, threshold function of forced denoising is used in signal filtering. It is shown from simulation and statistic analysis that the wavelet analysis technique is an effective method to eliminate noise from random drift signal of FOG, and that selection of wavelet base and decomposition scale also has an impact on filtering effect of wavelet analysis.

Keywords: FOG, Wavelet analysis, Forced denoising.

1. Introduction

FOG is a new type of solid-state gyroscope based Sagnac effect, with the advantage of all solid-state, high sensitivity, dynamic range, light weight, and low power, which is gradually replacing the traditional mechanical gyro and becoming a key component in inertial navigation system. Drift errors of FOG include constant drift errors and random drift errors. We can make use of the hardware and software on a drift test to compensate for constant drift errors; random drift errors therefore will be the main factors affecting the measurement accuracy. There are currently two main options used in signal filtering of FOG random drift errors; one option is using autoregressive (AR) model in time series analytical method to model FOG random drift signal mathematically, and then using Kalman filtering to remove random noise [1]; the other is directly using wavelet analysis method in signal processing of FOG random drift signal to suppress random noise [2].

In reality, the random drift signal is non-stationary and time-variable; therefore, it is quite difficult to establish accurate mathematical model to compensate for the random drift signal. AR model requires that the signal must be stable, normal and zero-mean, so FOG random drift signal is unable to satisfy all the demands of AR model and online modeling [3]. In this paper, we use wavelet analysis method in signal processing of FOG random drift signal. The simulation shows that wavelet analysis can effectively inhibit FOG random drift signal and improve FOG output accuracy; meanwhile, this method with simple algorithms also meets real-time requirements in FOG signal processing.

2. Introduction to Wavelet Analysis

As a new time-frequency analysis tool, wavelet analysis makes a feature of good localized nature in both the time domain and frequency domain. It is very suitable for analysis of non-stationary signals, known as mathematical microscope. Wavelet analysis has a number of important applications in many fields.

The so-called wavelet refers to the function family of wavelet base formed after scaling and translation.

\[ \psi_{a,b}(t) = \frac{1}{\sqrt{a}} \psi(t - \frac{b}{a}) \]  

There are several commonly used wavelet including Harr Wavelet, Daubechies wavelet, Biorthogonal wavelet, Coiflet wavelet, Morlet wavelet, Marr wavelet and so on. Wavelet transform of signal x(t) is defined as:
\[ W_r(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} f(t) \varphi(t-b/a) dt \]

Its inverse transform is:
\[ f(t) = \frac{1}{c_d} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} W_r(a,b) \frac{1}{\sqrt{a}} \varphi(t-b/a) db \frac{da}{a^2} \]

Compared with the traditional Fourier transform, wavelet transform can have a lower frequency resolution and a higher temporal resolution at high frequency portion of the signal; it also can have a higher frequency resolution and a lower temporal resolution at low frequency portion of the signal. Mallat algorithm enables fast algorithm based on wavelet transform to become a reality; the original signal may be decomposed at different decomposition scale by the tower algorithm so that we can obtain approximate coefficients at low frequency and detail coefficients at high frequency. Wavelet decomposition coefficients obtained from 3 decomposition scale are shown in Figure 1, in which
\[ S = A_1 + D_1 + D_2 + D_3 \]

![Wavelet decomposition scale 3 of signal](image)

Fig. 1. Wavelet decomposition scale 3 of signal.

### 3. Wavelet filtering principle

#### 3.1 Mathematical model of FOG random drift signal

FOG is the angular velocity sensor in attitude measurement system and its output signal contains the angular velocity information imposed on FOG as well as the angular velocity information of Earth’s rotation and all kinds of noise. Under conditions of stationary base, FOG measurement axis is perpendicular to the horizontal and the mathematical model of the FOG random drift signal is
\[ \omega(t) = \varepsilon_d + \varepsilon(t) + n(t) \]

Where, random constant drift \( \varepsilon_d \) is FOG output signal when the input angular rate is zero, which can be regarded as a constant in a short time; drift error \( \varepsilon(t) \) with strong randomness and weak cyclical trend is colored noise in FOG random drift signals; \( n(t) \) is zero-mean white noise\(^4\). According to the above formula, the parameters in the mathematical model are as follows: random constant drift \( \varepsilon_d = 0.1(\pi)/h \); colored noise \( \varepsilon(t) \) is simulated by a first order Markov process; the correlative time of Markov process is 20 second and the variance of Markov process is \( 0.01(\pi)/h^2 \); the variance of white noise \( n(t) \) is \( 0.04(\pi)/h^2 \). FOG random drift signal is shown in Figure 2.

![Random drift signal of FOG](image)

Fig. 2. Random drift signal of FOG.

#### 3.2 Filtering principle of FOG random drift signal

The purpose of filtering is to suppress the noise portion and recover the useful portion in the original signal. Typically, the useful signal in FOG random drift signal stays at a lower frequency band and random drift signal has the characteristics of low signal noise ratio, so the filtering process of FOG random drift signal is divided into the following three steps:

(a) To select an appropriate wavelet base according to issue’s need and determine the wavelet decomposition scale. To implement the wavelet decomposition of FOG random drift signal and obtain the wavelet coefficients at different decomposition scales.

(b) To select the appropriate thresholds and threshold functions for high frequency wavelet coefficients. To implement the threshold processing of wavelet coefficients for each decomposition scale.

(c) To combine low frequency wavelet coefficients and processed high frequency wavelet coefficients at different decomposition scales to complete the wavelet reconstruction of FOG random drift signal.

The selection of wavelet base, wavelet decomposition scale, wavelet threshold and threshold function will directly affect the de-noising effect of wavelet analysis in the filtering process of FOG random drift signal. Different wavelet bases have different characteristics in nature, so they will produce different filtering effect\(^5\). In practice, db wavelet is most selected to complete the signal decomposition and reconstruction. The larger the wavelet decomposition scale is, the easier to separate signal from noise and the better the filtering effect will be. However, once the wavelet decomposition scale is too large, a lot of useful signal will lost, which will result in large calculation.
errors and the increasing amount of computation. Therefore, an appropriate decomposition scale should be selected according to the actual accuracy need and various restrictions. Generally the wavelet decomposition scale takes a value from 4 to 6.

Common threshold functions include hard threshold function and soft threshold function. Let \( w_{jk} \) is the wavelet coefficients obtained from the wavelet decomposition, \( \lambda \) is the threshold, \( \hat{w}_{jk} \) is the wavelet coefficients processed by the threshold functions. The formulas of the hard-threshold function and soft threshold function are as follows:[6]:

\[
\hat{w}_{jk} = \begin{cases} 
  w_{jk} & |w_{jk}| \geq \lambda \\
  0 & |w_{jk}| < \lambda 
\end{cases} \quad (6)
\]

\[
\hat{w}_{jk} = \begin{cases} 
  \text{sign}(w_{jk})(|w_{jk}| - \lambda) & |w_{jk}| \geq \lambda \\
  0 & |w_{jk}| < \lambda 
\end{cases} \quad (7)
\]

Variance of the noise signal must be already known in hard threshold function and soft threshold function. For FOG random drift signal, noise variance is usually unknown[3]. In order to ensure the signal filtering is effective and timely, another threshold function-- forced denoising is used. The so-called forced denoising is to make the high frequency wavelet coefficients from the first decomposition scale to the last decomposition scale all equal zero, and then to use the low frequency coefficients of the last decomposition scale and the processed high frequency wavelet coefficients to reconstruct the original signal. Obviously, this threshold function doesn’t need the noise variance and its operation is relatively simple.

4. Simulation results and analysis

In this paper, db4 wavelet is used to implement the signal decomposition and reconstruction and the wavelet decomposition scale of 4. Threshold function of forced denoising is used in the simulation. The filtering results of FOG random drift signal in figure 2 are shown in figure 3 and table 1. Comparing figures 2 with figures 3, we find that not only the volatility of FOG random drift signal has been significantly reduced (gyro drift error is significantly reduced), but also the similar trend of the original signal is well maintained after wavelet filtering. In table 1, statistical characteristics of the original signal and the filtered signal are compared from the mean, the variance, the maximum and the minimum, respectively. The table indicates that wavelet filtering in processing of FOG random drift signal has received a good result.

The spectrum of the original signal and the filtered signal of FOG random drift signal is shown in figure 4 and figure 5, respectively. The results show that the spectrum of the original signal and the filtered signal is consistent within 1 Hz and that the amplitude spectrum of the filtered signal decreases rapidly in the high frequency band. This means the noise signal of FOG random drift signal has been effectively suppressed by the wavelet filtering. Useful signal of FOG random drift signal stays in the low frequency band, so the two figures show intuitive validity of wavelet analysis in FOG random drift signal filtering.

### Table 1. statistical characteristics between original signal and filtered signal.

<table>
<thead>
<tr>
<th></th>
<th>original signal</th>
<th>filtered signal</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean (°/h)</td>
<td>0.063</td>
<td>0.063</td>
</tr>
<tr>
<td>variance</td>
<td>0.050</td>
<td>0.012</td>
</tr>
<tr>
<td>maximum (°/h)</td>
<td>0.933</td>
<td>0.391</td>
</tr>
<tr>
<td>minimum (°/h)</td>
<td>-0.793</td>
<td>-0.278</td>
</tr>
</tbody>
</table>

![Fig. 3. Filtered signal of FOG](image3.png)

![Fig. 4. Amplitude spectrum of original signal](image4.png)

![Fig. 5. Amplitude spectrum of filtered signal](image5.png)
To analyze the filtering effect of the FOG random drift signal under different wavelet decomposition scales, we use db4 wavelet and threshold function of forced denoising to implement the wavelet decomposition and reconstruction. The results are shown in Table 2. With the decomposition scale increasing, the variance of the FOG random drift signal is smaller and wavelet filtering effect becomes better and better. According to the wavelet analysis theory, the greater the wavelet decomposition scale is, the lower the frequency resolution becomes. Therefore, we must determine a reasonable wavelet decomposition scale to satisfy the actual needs.

Table 2. Filtering effect among decomposition scales.

<table>
<thead>
<tr>
<th>mean (°/h)</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>variance</td>
<td>0.012</td>
<td>0.010</td>
<td>0.009</td>
</tr>
</tbody>
</table>

At present, the selection of the desired wavelet base in various engineering problems is usually determined by engineers’ experience and their constant test. We select db4, coif4, sym4 and bior1.5 wavelet bases commonly used in the literature to implement the wavelet decomposition and reconstruction of the original signal, respectively. Simulation results are shown in table 3 (the decomposition scale is 4 and the threshold function is forced denoising). The results show that different wavelet bases produce different filtering effect and db4 wavelet is the most suitable wavelet base in FOG random drift signal processing.

Table 3. Filtering effect among wavelet bases.

<table>
<thead>
<tr>
<th>mean (°/h)</th>
<th>db4</th>
<th>coif4</th>
<th>sym4</th>
<th>bior1.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>variance</td>
<td>0.012</td>
<td>0.013</td>
<td>0.013</td>
<td>0.014</td>
</tr>
</tbody>
</table>

5. Conclusions

FOG random drift signal is one of the key factors to affect the precision of navigation system. In this paper, wavelet analysis is used in FOG random drift signal processing and the threshold function of forced denoising is adopted. A comparative analysis of the statistical characteristics and the spectral characteristics is made between the original signal and the filtered signal. Simulation results show that wavelet analysis in FOG random drift signal filtering has a good noise reduction capability and that wavelet decomposition scale and wavelet base also have an impact on the filtering effect. The selection of the wavelet decomposition scale and wavelet base is usually determined by engineers’ experience and their constant test. Apparently, there is a lot of research work in theoretical aspects to be done in future.

References