Improvement of Reconstruction of Fourier Computer-generated Hologram Using Phase Freedom

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Abstract

In this paper, we referred to the simulation results for improving the resolution of the reconstructed image using a phase freedom on the original image in the interference type Fourier transform computer-generated hologram. We are able to arbitrarily specify the phase because the humans can not recognize the slight differences in the phase of the light. We can reconstruct the image more clearly and it can be expected to improve the resolution of the reconstructed image by reducing the errors between the reconstructed image and the original image using this phase freedom. To reduce the influences of the reference light in the prior study, we used a complex amplitude hologram. However, it is difficult to output the complex amplitude holograms and the creation of this hologram is not realistic. So, we created the hologram that can be output to the outside by adapting the proposed method to the interference type hologram that is relatively easy to be create. We must consider the effect of the reference light because of using the reference beam in the interference type hologram. Hereby, the practical hologram using the phase freedom can be generated.

Keywords:hologram, interference hologram, phase freedom

1. Introduction

A hologram is a product that is applied to the holographic techniques that is a technique for reconstructing the original three-dimensional image from the information recorded by the object beam from the three-dimensional object. The light has the amplitude and the phase information. The holograms can record this two information. The original three-dimensional images are reconstructed based on these amplitude and phase information. The computer-generated hologram is a method of calculating the complex amplitude

distribution of the light diffracted and transmitted from the object and reconstructs the image on the computer. By utilizing the computer-generated hologram, the recording of the complex amplitude information becomes easy. There is also an advantage that is able to create a hologram for the fictitious three-dimensional objects that do not exist in the world.

Generally, in order to reconstruct the image clearly, it is necessary to use a hologram that is sufficiently large. However, the size of the hologram is limited basically, so there is a limit of the image that can be reconstructed clearly. Therefore, in the previous study, we made sure that it is possible to reconstruct the image that is improved the resolution by using the phase freedom of the original image in the same size hologram.

In the previous study, we used the complex amplitude hologram that can be as much as possible to reduce the other errors to confirm the effectiveness of the phase freedom. However, it is difficult to output the complex amplitude hologram to the outside. We adapt this method to the interference hologram that is easy to create and output to the outside. It is a kind of amplitude hologram. The interference hologram is made by using the interference of the reference beam and the object beam. Therefore, the conjugate image is reconstructed along with the reconstructed image and it affects the reconstructed image. In this paper, we discuss the influence and the countermeasures of the conjugate image.

2. Interference Fourier transform hologram in computer-generated hologram

2.1 Errors of computer-generated hologram

The light has the amplitude and phase information. Hologram is possible to reconstruct the original three-dimensional image by recording them. Since the computer

Fig. 1. Optical system of the Fourier transform hologram.

handles only discrete values, a hologram that is created by this method includes the discrete and the quantization errors. The errors occur in the reconstructed image due to the influence of these errors, and this is the causes of the deterioration of the reconstructed image. And the hologram must be used sufficiently large size in order to record the all light because the light spreads in all directions. On the other hand, the new problems occur. These are the increase of the required memory space and the increase of the calculation time in the larger size hologram. Since there is an upper limit value of a certain hologram size, there is a limit of the size of the image that can be clearly reconstruct.

2.2 Interference type Fourier transform hologram

Fourier transform hologram is a hologram that is created by utilizing a Fourier transform action of the lens. We can get a Fourier transform action of the lens by placing the object that we want to record at the position of the front focus point of lens, whose focus length is defined as f . Interference Fourier transform hologram is obtained by recording the interference pattern that is obtained by interfering a reference beam and the Fourier transformed object beam utilizing the Fourier transformation action. And we can get a reconstructed image of the interference Fourier transform hologram by exposing a hologram to a plane wave and utilizing the Fourier transform action to the diffracted light again. Figure 1 shows the optical system of the Fourier transform hologram.

2.3 Computer-generated hologram

Fourier transform computer-generated hologram is created by calculating the object beam using the Fourier transform on a computer and obtaining the complex amplitude distribution on the hologram. There is an

advantage that is possible to calculate the object beam at a high speed because it is able to use a Fast Fourier Transform on a computer. The hologram distribution that are generated by computer are transcribed on the film by the super-resolution printer, or outputted to the outside by the spatial light modulator.

3. Iterative dummy area method and phase freedom

3.1 Iterative dummy area method

As described in 2.1, the errors occur in the reconstructed image due to the effects of the discretization, the quantization errors and the finite size of the hologram in the computer-generated hologram. In iterative dummy area method, we set the area whose initial value of the amplitude and phase around the input original image is zero. This area is called dummy area. We will reduce the errors of the reconstructed image by adjusting the amplitude and phase of the dummy area so that the errors of the reconstructed image are reduced. The technique for searching optimal solutions of the amplitude and phase of the dummy area in iteration, such as the errors of the reconstructed image is reduced, is iterative dummy area method. Figure 2 shows the flow of iterative dummy area method. At the same time, the reconstructed image becomes clear by optimizing the phase on the original image area so that the reconstructed image errors decrease. This is the use of the phase freedom.

3.2 Influence of interference by the reference beam

(a) Generation of interference hologram

The interference hologram is created by irradiating an unknown object beam that is propagating from the object and a known reference beam to interfere with the object beam on the hologram. The complex amplitude distribution of the wave front of the object beam on the hologram $O(x, y)$ is given by the following equation.

$$O(x, y) = |O(x, y)| \exp\{j\phi(x, y)\} \quad (1)$$

The complex amplitude distribution of the wave front of the

Fig. 2. Flow of repeat dummy area method.

reference beam to interfere on the hologram $R(x, y)$ is given by the following equation.

$$R(x, y) = |R(x, y)| \exp\{j\psi(x, y)\} \quad (2)$$

Then, the intensity distribution $I(x, y)$ of the interference fringes on the hologram is represented by the square of the sum of the complex amplitudes of the two beams.

$$I(x, y) = |O(x, y) + R(x, y)|^2 \quad (3)$$

Expand the (3) equation.

$$\begin{aligned} I(x, y) = & |O(x, y)|^2 + |R(x, y)|^2 \\ & + 2|O(x, y)||R(x, y)|\cos\{\psi(x, y) - \phi(x, y)\} \end{aligned} \quad (4)$$

The first term and the second term of this equation depends only on the intensity of each wave, and the third term depends on the relative phase of two waves. Therefore, it can be said that the amplitude and phase information of the object beam $O(x, y)$ are contained in the intensity distribution $I(x, y)$ of the interference fringe. We can create an interference hologram by recording the intensity distribution on the recording medium such as a photographic plate.

(b) Reconstruction of interference hologram

The hologram is reconstructed the amplitude and the phase of the object beam by irradiating the reference beam again. Assume that the amplitude transmittance $t(x, y)$ of the transparent film to create a hologram is the next equation

$$t(x, y) = t_0 + \beta T I(x, y) \quad (5)$$

t_0 is the constant background transmittance, β is a constant that is determined by the film material and development conditions and T is the exposure time. By (4) equation and the (5) equation,

$$\begin{aligned} t(x, y) = & t'_0 + \beta T |O(x, y)|^2 \\ & + \beta T |O(x, y)||R(x, y)| \exp\{-j\phi(x, y)\} \exp\{j\psi(x, y)\} \\ & + \beta T |O(x, y)||R(x, y)| \exp\{j\phi(x, y)\} \exp\{-j\psi(x, y)\} \\ = & t'_0 + \beta T |O(x, y)|^2 \\ & + \beta T R^*(x, y) O(x, y) \\ & + \beta T R(x, y) O^*(x, y) \end{aligned} \quad (6)$$

$t'_0 = t_0 + \beta T |R(x, y)|^2$ is a constant background transmittance and then $O^*(x, y)$ and $R^*(x, y)$ is the complex conjugate of $O(x, y)$ and $R(x, y)$. Also the reference light $R(x, y)$ is a plane wave.

If the film is irradiated by the same reference light, the complex amplitude distribution $g(x, y)$ of the reconstructed image is expressed by the following equation.

$$\begin{aligned} g(x, y) = & t(x, y) R(x, y) \\ = & t'_0 R(x, y) + \beta T R(x, y) |O(x, y)|^2 \\ & + \beta T |R(x, y)|^2 O(x, y) \\ & + \beta T |R(x, y)|^2 O^*(x, y) \end{aligned} \quad (7)$$

The first term of (7) equation is a direct transmission light of

(a) Noise

(b) Noise reduction

Fig. 3. Influence of the conjugate image.

the reference beam and the second term is the halo. The third term is the original object beam $O(x, y)$ and the fourth is the complex conjugate image of the original the object beam $O(x, y)$. The object beam is reconstructed from the hologram by the above ⁽¹⁾.

(c) Influence of the conjugate image

As described in 3.2 (b), the conjugate image is generated along with the reconstructed image in the interference hologram. Since this conjugate image is generated on the dummy area, it gives the noise to the reconstructed image through the dummy area. Figure 3 (a) shows this noise. The reconstructed image is deteriorated badly by the effects of this noise despite of using a dummy area and phase freedom.

Therefore, it is necessary to consider the influence of the conjugate image. The specific measures are as follows. It is possible to reduce the effect of this noise by the amplitude and phase on the dummy area of the conjugate image is zero in advance. In other words, this is that the dummy area on the conjugate image is not used.

4. Simulation and results

We simulated the two original images under the following conditions. In consideration of the effects of the conjugate image, we generate a hologram and simulated.

- Original images : 64×64 pixel (Figure 4)
- Dummy area : 4 times (256×256 pixel)
- Band limit : $1/2$
- Hologram : 256×256 pixel

The results of figure 5 are not used the iterative dummy area method and the initial phase is zero. That is not used the phase freedom on the original image. The results of figure 6 are used the dummy area method of 2000 times iteration and the initial phase is the random phase. That is used the phase

Fig. 4. Original image.

Fig. 5. Reconstruction image. (non-phase freedom)

Fig. 6. Reconstruction image. (phase freedom)

freedom on the original image. When we compare figure 5 and 6, in both F and Lenna reconstructed images, it can be seen that the results of figure 6 is clearer than the results of figure 5. Especially, in the Lenna image, the difference is obvious. The effectiveness of the phase freedom was shown also in the interference hologram by considering the effects of the conjugate image as with the complex amplitude hologram. Therefore, it becomes possible to generate a practical hologram with the phase freedom.

5. Conclusion

Generally, in order to reconstruct the image clearly, it is necessary to use a hologram that is sufficiently large size. However, the size of hologram is limited basically, so there is a limit of the image that can be reconstructed clearly. Therefore, we simulated the clear reconstructed image by using the phase freedom on the original image and improving

the resolution of the reconstructed image. To reduce the influence of the reference beam, we adapted the method to the complex amplitude hologram in the prior study. However, the complex amplitude hologram is not a practical hologram because the output to the outside is difficult. So we adapted the method to the interference hologram that is relatively easy to create. In this case, since the influence of the conjugate image may appear in the reconstructed image, it is necessary to create a hologram in consideration of the influence of the conjugate image.

In the simulation, by not using a dummy area on the conjugate image, it was possible to reduce the noise of the conjugate image. And we were able to confirm that the reconstructed image that was adapted the iterative dummy area method using a random phase in the initial phase has become clear. Accordingly, a technique for utilizing a phase freedom is also effective interference hologram. Thus the practical hologram using the phase freedom can be generated.

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