Parallel-connected type of Fibonacci Sequence Switched Capacitor DC-DC Converter

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Abstract

The Fibonacci sequence switched capacitor (SC) DC-DC converter is a high performance SC DC-DC converter in terms of power efficiency and circuit size among other SC DC-DC converters. However, there is room for improving the power efficiency of the Fibonacci sequence SC DC-DC converter. In this paper, the new topology of the Fibonacci sequence SC DC-DC converter is proposed. The new converter consists of two Fibonacci sequence SC DC-DC converters, and two converters are connected in parallel and across each other. Through this, the proposed converter can reduce internal resistance of it and then the proposed converter can improve power efficiency. To clarify the characteristic of the proposed converter, theoretical analysis and integrated circuit emphasis (SPICE) simulations of the proposed converter are performed.

Keywords: DC-DC converter, Fibonacci sequence, switched capacitor.

1. Introduction

With the importance of energy harvesting system, photovoltaic system, electric vehicles, hybrid vehicles and so on, the power converter of those devices is required to have high power efficiency and small size. For this reason, switched capacitor (SC) DC-DC converter was suggested.

An SC DC-DC converter consists of switches and capacitors; the switches are semiconductor switch like metal-oxide semiconductor field-effect-transistor (MOSFET) or transistor. The first idea of SC DC-DC converter was proposed as SC voltage multiplier⁽¹⁾. From the first idea, many types of the SC DC-DC converter were suggested and designed. Among others, the Fibonacci sequence SC DC-DC converter can meet the demand of high power efficiency and

small size. It was confirmed in previous researches that the Fibonacci sequence SC DC-DC converter has the highest power efficiency and the smallest size among voltage equational type, series-parallel type, ring type, multistage switched capacitor voltage multiplier⁽²⁾⁻⁽⁶⁾. However, there is room for improving the Fibonacci sequence SC DC-DC converter about power efficiency and operating state.

The conventional Fibonacci sequence SC DC-DC converters operated by using three states: two charging states and one discharging state. The extra discharging state of the conventional converter makes the converter's impedance higher. In this situation, we suggest the new topology of Fibonacci sequence SC DC-DC converter in this paper. The new topology has two states for operating it by connecting two Fibonacci sequence SC DC-DC converters in parallel and across each other. Through this, the proposed converter can operate by two states, which means that internal resistance of the converter is decreasing and improve the power efficiency of it.

The rest of this paper is organized as follows. In chapter 2, the circuit configuration of the conventional and the proposed Fibonacci sequence SC DC-DC converter is presented. In chapter 3, the comparison between the conventional and the proposed Fibonacci sequence SC DC-DC converter is presented by the theoretical the analysis. In chapter 4, the characteristic of the proposed converter is clarified by SPICE simulation. Finally, conclusion and future study are shown in chapter 5.

2. Circuit Configuration

2.1 Conventional Fibonacci Sequence SC DC-DC converter

Fig. 1 shows the conventional Fibonacci sequence DC-DC converter. The converter consists of 10 transistor

Fig. 1. Conventional Fibonacci sequence SC DC-DC converter.

Table 2. Switching rule of the conventional Fibonacci sequence SC DC-DC converter.

Step	Switch to turn on	Switch to turn off
State-1	S_0, S_4, S_5	Others
State-2	S_1, S_2, S_3, S_7, S_8	Others
State-3	S_1, S_3, S_6, S_9	Others

switches and 4 capacitors. The converter operate by the switching rule as shown on table 1. The reason that the converter's title has Fibonacci is that the capacitors in the converter are charged by the following equation (1).

$$V_{Cn} = V_{Cn-1} + V_{Cn-2} \text{ where } n=\{3\} \quad (1)$$

with $V_{C1}=V_{in}$ $V_{C2}=V_{C1}+V_{in}$

In State-1, the converter charges the even number capacitor (C_2) up to 2 times of the input voltage. In State-2, the odd number capacitors (C_1, C_3) are charged up to 1 times and 3 times of the input voltage respectively. In State-3, the charged capacitors (C_2 and C_3) and the input source generate 6 times stepped-up voltage. In other words, State-1 and -2 are for charging the capacitors and State-3 is for discharging the capacitors' energy to the output. Fig. 2 shows the waveforms of the current and voltage for each state.

2.2 Proposed Fibonacci sequence SC DC-DC converter

The idea of the proposed Fibonacci sequence SC DC-DC converter was starting from room for improving the conventional converter as following. 1) A possibility to decrease the number of the operating states. 2) A possibility to improve the power efficiency of the converter. The

Table 2. Switching rule of the conventional Fibonacci sequence SC DC-DC converter.

Step	Turn on	Turn off
State-1	$S_0, S_4, S_5, S_9, S_8, A_1, A_2, A_3, A_6, A_7$	Others
State-2	$A_0, A_4, A_5, A_9, A_8, S_1, S_2, S_3, S_6, S_7$	Others

Fig. 2. Waveforms of Conventional Fibonacci sequence SC DC-DC converter. (a) Current of switches. (b) Current and voltage of capacitors.

solution is to remove some switches of the conventional converter and then to connect two converters in parallel and cross. In this way, it is possible to operate the proposed converter without the state-3 of the conventional converter.

The proposed converter consists of two conventional Fibonacci sequence SC DC-DC converter with removing the switch S_6 in the conventional converter as shown fig. 1. The two converters are connected in parallel and cross by using four transistors switch as shown fig 3.

Fig. 3. Proposed Fibonacci sequence SC DC-DC converter.

(a)

(b).

Fig. 4. Waveforms of proposed Fibonacci sequence SC DC-DC converter. (a) Current of switches. (b) Current and voltage of capacitors.

Fig. 5. Four-terminal equivalent model for the converter in this paper.

Unlike the conventional converter of fig. 1, the proposed converters operated by two states as shown on table 2. For State-1 and -2, each cell of the proposed converter charges and discharges such as fig. 4. Each cell's charging way is the same as that of the conventional converter as above-mentioned. In a steady state, the proposed converter has C3 and C6 charged to 3 times of the input voltage respectively. C3 and C6 can generate 6 times stepped-up voltage to connect two capacitor's node in series, which means that it is not necessary to use the extra operating state for discharging like State-3 of the conventional converter. Namely, the proposed converter can operate charging and discharging at the same time.

3. Theoretical Analysis

3.1 Equivalent model

To analyze theoretically the converters, the four-terminal equivalent model is used. Fig. 5 describes the model, where n means the ratio of conversion and R_{sc} is the switched capacitor resistance. The value of R_{sc} is only one component to consume power from the input source in the model. Therefore, the value of R_{sc} has influence on power efficiency of the converter. By using circuit laws and R_{sc} value, it is implemented to derive the power efficiency, the relation with the input current and the output current and the relation with the input voltage and the output voltage in the theoretical analysis.

In the theoretical analysis, we assume two things as follows. 1) All switches in the converters are an ideal switch with on-resistance R_{on} . 2) Time constant related with capacitor and resistance is much larger than the system clock of the converters.

3.2 Theoretical analysis of the conventional Fibonacci

Fig. 6. Instantaneous equivalent circuit of conventional converter at each state

Sequence SC DC-DC converter

In the theoretical analysis of the conventional converter, the interaction relations are based on the instantaneous equivalent circuits shown in fig. 6.

In a steady state, all of the differential value of the electric charge satisfy equation (2).

In equation (2), Δq_{Ti}^k means that the electric charge of the k -th capacitor in State- i , and T is the total period of the system clock for State-1, 2 and 3. Additionally, T_i is the i -th period of the State- i . By applying the Kirchhoff's current law to the each circuit in fig 6, the interaction relations can be derived. Equations (3) - (5) are for State-1, 2 and 3 respectively.

$$\Delta q_{T_1, V_{in}} = -\Delta q_{T_1}^1, \Delta q_{T_1}^3 = 0 \quad (3)$$

$$\text{and } \Delta q_{T_1, V_o} = \Delta q_{T_1}^4$$

$$\Delta q_{T_2, V_{in}} = \Delta q_{T_2}^1 - \Delta q_{T_2}^2, \Delta q_{T_2}^2 = -\Delta q_{T_2}^3 \quad (4)$$

$$\text{and } \Delta q_{T_2, V_o} = \Delta q_{T_2}^4$$

$$\Delta q_{T_3, V_{in}} = -\Delta q_{T_3}^2, \Delta q_{T_3}^1 = 0 \quad (5)$$

$$\text{and } \Delta q_{T_3, V_o} = \Delta q_{T_3}^4 + \Delta q_{T_3}^3$$

By using the equations (3) – (5), the average value of the input current and output current can be derived as the equation (6).

$$\bar{I}_{in} = \frac{\sum_{i=1}^3 \Delta q_{T_i, V_{in}}}{T} = \frac{\Delta q_{T, V_{in}}}{T} \quad (6)$$

and

$$\bar{I}_o = \frac{\sum_{i=1}^3 \Delta q_{T_i, V_o}}{T} = \frac{\Delta q_{T, V_o}}{T}$$

By substituting equations (2) - (5) into equation (6), the relation between the input current and the output current can be obtained as equation (7).

$$\bar{I}_{in} = -6\bar{I}_o, \quad (7)$$

where $\Delta q_{T_i, V_{in}} = -6\Delta q_{T_i, V_o}$ and $n = 6$

Next, the calculation of the consumed energy for each state is necessary to derive the value of R_{sc} . The total consumed energy can be expressed as equation (8) by using equations (2) - (5).

$$W_T = \sum_{i=1}^3 W_{T_i} = \frac{96R_{on}}{T} (\Delta q_{V_o})^2, \quad (8)$$

$$\text{where } W_{T_1} = \frac{12R_{on}}{T_1} (\Delta q_{V_o})^2,$$

$$W_2 = \frac{16R_{on}}{T_2} (\Delta q_{V_o})^2 \text{ and}$$

$$W_3 = \frac{4R_{on}}{T_3} (\Delta q_{V_o})^2$$

The consumed energy of the equivalent model shown in fig. 5 is able to express as equation (9).

$$\sum_{i=1}^3 \Delta q_{T_i}^k \text{ where } T = T_1 + T_2 + T_3 \quad (2)$$

$$\text{and } T_1 = T_2 = T_3 = \frac{T}{3}.$$

$$W_T = \left(\frac{\Delta q_{V_o}}{T} \right)^2 \cdot R_{sc} \cdot T \quad (9)$$

By comparing equation (8) and (9), the value of R_{sc} is obtained as equation (10).

$$R_{sc} = 96R_{on} \quad (10)$$

The power efficiency η and the output voltage V_o are

can be obtained as equation (11) by using the value of R_{sc} and the equivalent model.

$$\eta = \frac{R_L}{R_L + 96R_{on}} \quad \text{and} \quad V_o = \left(\frac{R_L}{R_L + 96R_{on}} \right) \times 6V_{in} \quad (11)$$

3.3 Theoretical analysis of the proposed Fibonacci Sequence SC DC-DC converter

The interaction relations of the proposed converter are based on the instantaneous equivalent circuits shown in fig. 7.

In steady state, all of the differential value of the electric charge satisfy equation (12).

$$\sum_{i=1}^2 \Delta q_{T_i}^k \quad \text{where } T = T_1 + T_2 \quad (12)$$

and $T_1 = T_2 = \frac{T}{2}$.

Each parameter of $\Delta q_{T_i}^k$ is used in the same way as above-mentioned at section 3.2. The interaction relations for each equivalent circuit in fig. 7 can be derived as equation (13) and (14) by using Kirchhoff's current law.

$$\Delta q_{T_1, V_{in}} = -\Delta q_{T_1}^1 + \Delta q_{T_1}^4 - \Delta q_{T_1}^5, \quad \Delta q_{T_1}^3 = -\Delta q_{T_1}^2, \Delta q_{T_1}^5 = -\Delta q_{T_1}^6 + \Delta q_{T_1}^3 \quad (13)$$

$$\text{and } \Delta q_{T_1, V_o} = \Delta q_{T_1}^7 + \Delta q_{T_1}^3$$

$$\Delta q_{T_2, V_{in}} = -\Delta q_{T_2}^4 + \Delta q_{T_2}^1 - \Delta q_{T_2}^2, \quad \Delta q_{T_2}^4 = -\Delta q_{T_2}^5, \Delta q_{T_2}^2 = -\Delta q_{T_2}^3 + \Delta q_{T_2}^6 \quad (14)$$

$$\text{and } \Delta q_{T_2, V_o} = \Delta q_{T_2}^7 + \Delta q_{T_2}^6$$

The average value of the input current and output current can be derived as the equation (15) by using equations (13) and (14).

$$\overline{I_{in}} = \frac{\sum_{i=1}^2 \Delta q_{T_i, V_{in}}}{T} = \frac{\Delta q_{T, V_{in}}}{T} \quad \text{and} \quad (15)$$

$$\overline{I_o} = \frac{\sum_{i=1}^2 \Delta q_{T_i, V_o}}{T} = \frac{\Delta q_{T, V_o}}{T}$$

By substituting equations (13) and (14) into equation (15), we can confirm that the relation between the input current and the output current is same as equation (7).

Next step is the calculation of the consumed energy for

Fig. 7. Instantaneous equivalent circuit of proposed converter at each state

each state in order to derive the value of R_{sc} . The total consumed energy can be expressed as equation (16) by using equations (12) - (14).

$$W_T = \sum_{i=1}^2 W_{T_i} = \frac{20R_{on}}{T} (\Delta q_{V_o})^2, \quad (16)$$

$$\text{where } W_{T_1} = W_2 = \frac{10R_{on}}{T_1} (\Delta q_{V_o})^2$$

By comparing equation (9) and (16), the value of R_{sc} is obtained as equation (17).

$$R_{sc} = 20R_{on} \quad (17)$$

The power efficiency η and the output voltage V_o are can be derived as equation (18) by using the value of R_{sc} and the equivalent model.

Table 3. Value of R_{sc} and the number of circuit components

Parameter	Conventional	Proposed
R_{sc}	$96R_{on}$	$20R_{on}$
Capacitors' umber	5	7
Switches' number	10	18

Table 4. Simulation conditions

Parameter	Value
V_{in}	12V
Capacitance	10 μ F
R_{on}	0.1 Ω
R_L	10 Ω ~10k Ω
T	1.5 μ sec

$$\eta = \frac{R_L}{R_L + 20R_{on}} \quad \text{and} \quad (18)$$

$$V_o = \left(\frac{R_L}{R_L + 20R_{on}} \right) \times 6V_{in}$$

3.4 Comparison between the conventional converter and the proposed converter

Through the theoretical analysis, we can know that the value R_{sc} of the proposed converter has 4.8 times lower than that of the conventional converter. It means that the proposed converter can improve the power efficiency of it. Table 3 shows the value of R_{sc} and the number of circuit component respectively.

4. Simulation

To clarify the characteristics of the converters, the proposed and the conventional converter are simulated with the conditions as table 4. Additionally, the reason that the range of load resister starts from 10 Ω is to simulate the performance of the converters at the worst power efficiency. As you can see from fig. 8, the proposed converter offers higher output voltage than the conventional converter. The reason for the difference of output voltage of two converters is each power efficiency. Fig. 9 demonstrates that the proposed converter improves the power efficiency over whole output power by comparing the conventional converter. Concretely, the power efficiency of the proposed converter can improve about 30% higher than the conventional converter when the output power is about 130W.

(a) Efficiency as the load increased.

(b) Efficiency as the output power increased.

Fig. 9. Simulated and theoretical power efficiency.

(a) $R_L=10 \Omega$.

(b) $R_L=50 \Omega$.

Fig. 8. Simulated output voltage with the load.

5. Conclusion

A new topology of Fibonacci sequence SC DC-DC converter has been proposed in this paper. The proposed SC DC-DC converter can operate by using two states, whereas the conventional Fibonacci sequence SC DC-DC converter operates by three states. While the circuit size of the

conventional converter is smaller than the proposed converter, we confirmed theoretically that the proposed converter has higher power efficiency. The result of the simulations showed that the proposed converter operated with higher power efficiency than the conventional converter. The proposed converter can operate with the maximum 30% higher power efficiency at about 130W of the output power than the conventional converter.

The experiment of the proposed converter are left as a future study in order to confirm the validity of it.

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